Was Javert right to be suspicious?
Unpacking treatment effect heterogeneity of alternative sentences on time-to-recidivism in Brazil [*]

Santiago Acerenza[†]   Vitor Possebom[‡]   Pedro H. C. Sant'Anna[§]

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Abstract

This paper presents econometric tools to unpack the treatment effect heterogeneity of punishing misdemeanor offenses on time-to-recidivism. We show how one can identify, estimate, and make inferences on the distributional, quantile, and average marginal treatment effects in setups where the treatment selection is endogenous and the outcome of interest, usually a duration variable, is potentially right-censored. We explore our proposed econometric methodology to evaluate the effect of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, between 2010 and 2019, leveraging the as-if random assignment of judges to cases. Our results highlight substantial treatment effect heterogeneity that other tools are not meant to capture. For instance, we find that people whom most judges would punish take longer to recidivate as a consequence of the punishment, while people who would be punished only by strict judges recidivate at an earlier date than if they were not punished.

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[†]Universidad ORT Uruguay. Email: acerenza@ort.edu.uy
[‡]Sao Paulo School of Economics - FGV. Email: vitor.possebom@fgv.br
[§]Emory University. Email: pedro.santanna@emory.edu

To owe his life to a malefactor, to accept that debt and to repay it; to be, in spite of himself, on a level with a fugitive from justice, and to repay his service with another service; to allow it to be said to him, "Go," and to say to the latter in his turn: "Be free"; to sacrifice to personal motives duty, that general obligation, and to be conscious, in those personal motives, of something that was also general, and, perchance, superior, to betray society in order to remain true to his conscience; that all these absurdities should be realized and should accumulate upon him,—this was what overwhelmed him.

Les Misérables by Victor Hugo

1   Introduction

Understanding how different types of sanctions impact defendants' behavior is a critical area of research among Crime Economists. For misdemeanors, which are relatively minor offenses, we know relatively little about the causal effects of prosecution on defendants' subsequent criminal justice involvement (Agan, Doleac and Harvey, 2023), and arguably even less about the effect of alternative sentences on defendants' recidivism.[1] This is a particularly important topic, as a misdemeanor charge is often the point of entry of individuals to the criminal justice system. If they are convicted, they will then acquire a criminal record. This could "lower the cost" of committing other crimes, or work as intended and prevent future criminal behavior. In practice, it is unclear which direction dominates, and it is likely that this varies from individual to individual. Being able to understand the types of defendants that are on either side is therefore desirable and policy-relevant.

In this article, we propose econometric tools that are tailored to highlight treatment effect heterogeneity with respect to the unobserved punishment resistance on time-to-recidivism. These tools can then be used to shed light on to whom punishments are working as intended in terms of avoiding (or postponing) recidivism. Importantly, our tools account for the fact that (i) time-to-recidivism is a duration outcome that is subject to right-censoring, i.e., not all defendants recidivate by the end of the sampling period (but may do it later on); (ii) treatment selection is endogenous and judges are likely to have more information about the case than researchers; (iii) individuals may be inherently heterogeneous (essential heterogeneity); (iv) one may be interested in causal effects beyond local average treatment effect parameters; (iv) distributional features of time-to-recidivism may also be relevant.

We achieve these goals by extending the marginal treatment effects (MTE) framework of Heckman and Vytlacil (1999, 2005) to setups in which the outcome variable is right-censored.

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[1]See Huttunen, Kaila and Nix (2020), Giles (2023), Klaassen (2021), Possebom (2023), and Lieberman, Luh and Mueller-Smith (2023) for some advances in this area.

The main requirement to use our tools is having access to a continuous instrument such that the propensity score has large support.[2] In the context of Crime Economics, this instrument is usually given by the trial judge's leniency rate. Our tools can be used to recover various heterogeneity-rich causal effects, including distributional and quantile MTE (Carneiro and Lee, 2009). Indeed, when we use our MTE tools to study the impact of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, we find substantial treatment effect heterogeneity that traditional causal inference tools are not meant to capture.

We find the MTE framework particularly attractive to studying the effect of punishments on time-to-recidivism. For example, it allows one to assess the treatment effect of punishment on recidivism for defendants on a margin of indifference between being punished or not. By considering different degrees of unobserved punishment resistance, the MTE provides a detailed picture of how punishments heterogeneously affect recidivism and can be used to design better sentencing criteria and/or train judges to follow a specific protocol. For example, suppose that one finds a negatively sloped MTE function with some positive and negative effects. This would suggest that defendants who would be punished even by very lenient judges — i.e., defendants with low unobserved punishment resistance — would take more time to recidivate as a result of the punishment (punishment is working as intended). On the other hand, defendants who would be fined only by very strict judges — i.e., defendants with high unobserved punishment resistance — would recidivate sooner than if they were not punished (punishment is not effective, perhaps because of scaring effects of a criminal record). Such degree of heterogeneity is usually washed out when using single summaries of treatment effects such as local average treatment effect (LATE) (Imbens and Angrist, 1994).[3]

Our methodological results highlight that extending the MTE framework to deal with time-to-recidivism (or a duration variable subject to right-censoring) introduces some interesting challenges depending on the censoring mechanism. For instance, if censoring is independent of potential outcomes, we can point-identify the distributional marginal treatment effect (DMTE) and quantile marginal treatment effect (QMTE) functions for some, but not necessarily all, support points and quantiles. Nonparametrically identifying the entire DMTE and QMTE functions is only possible if the support of the censoring variable is at least as large as the support of the duration outcome.[4] We address these challenges and propose semiparametric

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[2]See Brinch, Mogstad and Wiswall (2017) and Mogstad, Santos and Torgovitsky (2018) for extensions of the MTE framework that does not require this support condition.

[3]To be fair, we stress that LATE was not meant to highlight this degree of heterogeneity and that it has the advantage of only requiring binary instruments. However, when the empirical setting presents a continuous instrument, the definition of a "complier" is less clear than in the binary instrument case, potentially making the LATE results more challenging to interpret formally.

[4]When this restriction is not satisfied, one can nonetheless nonparametrically point-identify truncated MTE functions, which are still well-defined causal parameters. See Appendix H for more details.

estimators and inference procedures for the DMTE and QMTE functions and establish their large sample properties. We also discuss two potential avenues to handle cases where censoring possibly depends on the potential outcomes. First, we leverage a negative regression dependence between potential outcomes and censoring variables (Lehmann, 1966), which can be justified when defendants commit fewer crimes over time. Second, we discuss how one can continuously relax the independent censoring assumption. Both of these strategies that do not impose exogenous censoring mechanisms lead to partial identification of the causal parameters.

As the discussion above highlights, explicitly dealing with (right-censored) time-to-recidivism outcomes can be challenging. One question that naturally arises is whether empirical researchers can bypass these challenges and use standard causal inference tools. One potential way to avoid such challenges that some empirical researchers have considered is to impose some sample construction restrictions and focus on recidivism within a given time frame, say two years. This essentially changes the outcome of interest from time-to-recidivism to whether one recidivates within 2 years. The censoring issue is avoided if one follows *all* defendants for at least two years. Although this is convenient and generically valid, this procedure has drawbacks: (a) it changes the question of interest by changing the outcome variable, and (b) the choice of the cutoff (two years in this example) is arbitrary. For instance, it may be that punishment has no effect on recidivism within two years but then has an effect within three years (or one year). These concerns are minor if one is interested only in recidivism within a known time frame. One potential way to assess whether this is the case is to ask ourselves: if censoring was not a concern, would we use time-to-recidivism or recidivism within two years as the outcome? If the answer is only the latter, then standard practice is justified. If not, we caution researchers that the conclusions of their analysis may be sensitive to the cutoff used for "binarization" of the time-to-recidivism outcome and that directly using the time-to-recidivism outcome may lead to different conclusions. See Appendix E.2 for a simple example of this.

If the conclusions of a study might be sensitive to the cutoff used for binarization, a natural next step would be for empirical researchers to consider a handful of cutoffs and show that the results are "robust" to the cutoff used. We note that this happens often in practice. However, when doing this, researchers often ensure that the sample used for the entire analysis is not contaminated with compositional changes, which can lead to loss of power for shorter horizons. To be more concrete, suppose that researchers considered three thresholds for the "binarization": two, three, and five years. To ensure that the same defendants are analyzed according to all transformed outcomes, researchers commonly restrict the sample to contain only the defendants we have observed in the data for at least five years. As such, they drop available observations for the shorter horizons, leading to a loss of statistical power.

This discussion leads to the next natural question: what if we considered a large set of

cutoffs and did not restrict the sample across cutoffs? Heuristically, one can interpret our distributional MTE results as doing precisely that—avoiding choosing arbitrary cutoffs and considering recidivism within y periods for a continuum of $y \in \mathbb{R}_+$. Our quantile MTE results "transform" our distributional MTE results so the underlying treatment effects are expressed in the same units as the time-to-recidivism outcome, leading to additional insights. Here, though, it is worth stressing that we explicitly tackle the censoring problem when considering the continuum of cutoffs by adapting the Frandsen's (2015) reweighing approach to our context. Erroneously ignoring the censoring problem can indeed lead to misleading conclusions.

We show the appeal of our causal inference tools by evaluating the effect of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, between 2010 and 2019.[5] Our treated group (punished group) contains the defendants who were fined or sentenced to community services, and our untreated group (unpunished group) contains defendants who were acquitted or whose cases were dismissed.[6] To measure recidivism, we check whether the defendant's name appears in any criminal case within the sample period after the final sentence's date. More precisely, our outcome variable is the time between the final sentence and a subsequent criminal case. Since the sampling period is finite, the outcome variable is right-censored.

To deploy our proposed methodology, we need a continuous instrumental variable since we face endogenous selection into punishment.[7] We use the trial judge's leave-one-out rate of punishment (or "leniency rate") as an instrument for the trial judge's decision (Bhuller, Dahl, Loken and Mogstad, 2020; Agan et al., 2023). Importantly, this instrumental variable is continuous with large support and is independent of the defendant's counterfactual criminal behavior because judges are randomly assigned to cases conditional on court districts according to state law in São Paulo. Our outcome data — time-to-recidivism — is right-censored by construction, requiring a methodology that accounts for this identification challenge.

Empirically, we find that the cross-district average QMTE functions for .10, .15, .25, .40, .50, and .75 quantiles are heterogeneous with respect to unobserved punishment resistance. The treatment effects are estimated to be sometimes positive and sometimes negative. More pre-

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[5]São Paulo is the largest state in Brazil, with a population above 44 million people according to the Brazilian Census in 2022. Moreover, analyzing the impact of judicial policies on criminal behavior in this state is relevant due to its relatively high criminality. For example, according to São Paulo Public Safety Secretary, there were 12.56 murders, 1261.95 thefts, and 498.82 robberies per 100,000 inhabitants in 2023. Importantly, theft is one of the most common crimes in our sample.

[6] Our sample only contains cases whose punishment must be a fine or community service sentence. In 1998, a new law established that criminal charges whose maximum prison sentence is less than four years in the 1940 Criminal Law Code must be punished with a fine or a community service sentence from that point onwards. As we are particularly interested in the effect of fines and community service sentences, we focus on these specific criminal cases and define them as misdemeanor offenses.

[7] If the researcher is comfortable with parametric assumptions, it is possible to use a discrete instrument as suggested by Brinch et al. (2017).

cisely, we find that people who would be punished by most judges (those with low punishment resistance) take longer to recidivate as a consequence of the punishment, while people who would be punished only by strict judges (high punishment resistance) recidivate at an earlier date than if they were not punished. This result suggests that designing sentencing guidelines that encourage strict judges to become more lenient could increase time-to-recidivism.

Lastly, we compare our results with methods that ignore that time-to-recidivism is right-censored. We find attenuated effects when ignoring the censoring problem and estimating the QMTE model semiparametrically. If one used IV quantile regressions (ignoring censoring), one would find that treatment effects are slightly negative but would not be able to highlight heterogeneity as in the QMTE functions. These differences highlight that our tools can indeed bring new insights to policy discussions.

Related literature: This article contributes to different branches of literature. Concerning its theoretical contribution, our work contributes to the literature on MTE by extending the MTE framework of Heckman and Vytlacil (1999, 2005), Heckman, Urzua and Vytlacil (2006), and Carneiro and Lee (2009) to a setting with right-censored data. We also contribute to the literature on duration outcomes; see, e.g., Khan and Tamer (2009), Frandsen (2015), Tchetgen Tchetgen, Walter, Vansteelandt, Martinussen and Glymour (2015), Sant'Anna (2016, 2021), Delgado, Garcia-Suaza and Sant'Anna (2022). None of these papers consider MTE-type parameters as we do. Among these, the closest work to ours is Frandsen (2015), which considers the case where the censoring variable is observed and shows how one can identify distributional and quantile local treatment effects, assuming that censoring is exogenous. Our results can be interpreted as an extension of Frandsen (2015) to the MTE framework, possibly allowing for endogenous censoring.

Concerning its empirical contribution, our work is inserted in the literature about the effect of fines and community service sentences on future criminal behavior; see, e.g., Huttunen et al. (2020), Giles (2023), Klaassen (2021), Possebom (2023), and Lieberman et al. (2023).[8] They all focus on binary variables indicating recidivism within a pre-specified period. Within these, as we build on his dataset, Possebom (2023) is the closest to ours. However, his focus differs greatly from ours, and he does not handle duration outcomes as we do.

Organization of the paper: The rest of the paper is organized as follows. Section 2 describes the data and causal parameters and discusses why focusing on time-to-recidivism is useful in our empirical application. Section 3 presents our model, discusses our identifying assumptions, and provides our identification results with a right-censored outcome variable.

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[8]This literature focuses on non-incarceration punishments for individuals who are already being prosecuted. The reader who is interested in the effect of alternative solutions to prosecution may check the recent work developed by Agan et al. (2023) and Shem-Tov, Raphael and Skog (2024). Readers interested in incarceration's effect may check the recent work of Rose and Shem-Tov (2021), Humphries, Ouss, Stavreva, Stevenson and van Dijk (2023) and Kamat, Norris and Pecenco (2023).

Section 4 explains how to semiparametrically estimate the objects necessary to implement the identification strategy described in the previous section. Finally, Section 5 discusses the empirical results, while Section 6 concludes and describes other possible applications for our proposed methods.

This paper also contains an online supporting appendix. Our main identification proofs are detailed in Appendix A. Appendix B derives the asymptotic distribution of our semiparametric estimators. In Appendix C, we summarize how we constructed our dataset. Additional empirical results can be found in Appendix D. Appendix E provides two arguments that justify focusing on the MTE function of duration outcomes. Appendix F identifies a conditional version of our causal parameters under weaker assumptions than the ones used in the main text. Appendix G proposes alternative partial identification strategies. Finally, Appendix H discusses the average marginal treatment effect function.

2 Empirical context, data, and causal parameters

We study the effect of alternative sentences in the form of fines and community service on time-to-recidivism in the state of São Paulo, Brazil. Towards this end, we collect data from all criminal cases brought to the Justice Court System in the State of São Paulo, Brazil, between January 4th, 2010, and December 3rd, 2019.[9] According to a Brazilian law from 1998, criminal charges whose maximum prison sentence is less than four years in the 1940 Criminal Law Code must, from that year onwards, be punished with a fine or a community service sentence if the defendant is found guilty. As we are particularly interested in the effect of these alternative sentences, we focus on these specific criminal cases and define them as misdemeanor offenses. We also restrict our sample to cases that started between 2010 and 2017. Based on these restrictions, the most common types of crime in our sample are theft and domestic violence.

There are 332 court districts in the state of São Paulo. Criminal complaints are analyzed by a trial judge working at the court with geographic jurisdiction over the location of the alleged offense.[10] Moreover, there are 862 trial judges during our sample period. We keep 642 judges who analyzed more than 20 cases out of these. In court districts that have more than one judge, the case is randomly allocated to one of the judges according to a computer algorithm that creates a lottery of judges. Of the 332 court districts in our sample, 193 have more than

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[9]See Appendix C for an overview of the data-construction.

[10] In Brazil, instead of being elected, judges are appointed for life based on their performance in a civil service exam and frequently serve as judges until retirement (Laneuville, 2024). Furthermore, they make decisions about conviction, sentence type and sentence intensity in all cases but "crimes against life" (murder, attempted murder, manslaughter, incentivizing or assisting suicide, and abortion). When a person is accused of a "crime against life", a jury decides whether the person is guilty while the presiding judge makes decisions related to incarceration length. Importantly, our sample does not contain "crime against life" cases. Therefore, all our cases are entirely decided by appointed judges.

one judge who analyzed more than 20 cases. Given that our econometric procedure explores the random allocation of judges to criminal cases and their different leniency levels, we restrict our attention to court districts with more than two judges who analyzed 20 cases or more. After imposing these two restrictions, our sample has 525 trial judges from 193 different court districts, handling 43,468 cases in total.

Figure 1 shows the distribution of the number of judges for every court district in São Paulo (Figure 1a) and the distribution of the number of cases for every judge in São Paulo (Figure 1b). The average number of cases per judge is 120 in our full sample and 159 after we focus on judges who analyzed more than 20 cases. Furthermore, the average number of judges per court district is 2.2 in the full sample and 3.0 in our restricted sample.

Figure 1: Descriptive Statistics for the Number of Judges and the Number of Cases
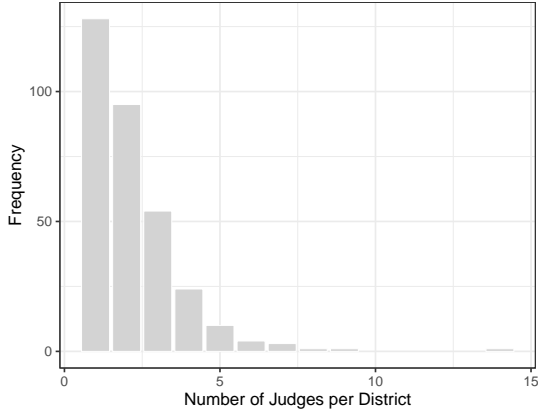
(a) Histogram of the Number of Judges per Court District
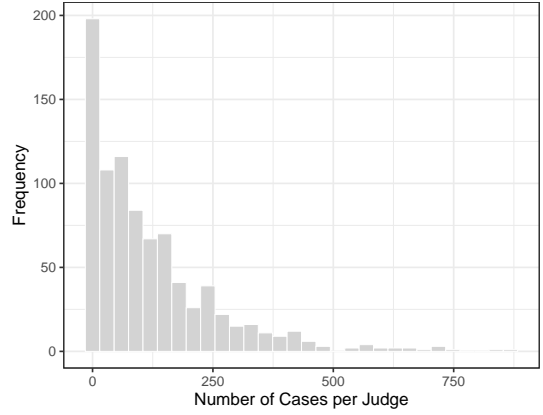
(b) Histogram of the Number of Cases per Trial Judge

Notes: Figure 1a plots the histogram of the number of judges per court district in our full sample, while Figure 1b plots the histogram of the number of cases per trial judge in our full sample.

2.1 Defining the variables of interest

In our dataset, we observe the defendant's full name, the defendant's court district, the case's starting date, the assigned trial judge's full name, the case's final ruling, and the case's final ruling's date. All our variables of interest will be constructed from these pieces of information. Henceforth, let X denote the full set of court district dummies, which will play the role of covariates in our analysis.

Let us start with our treatment variable, D, which denotes the final ruling in the case. Defendants who were fined or sentenced to community services because they were either convicted or signed a non-prosecution agreement according to the final ruling in their case belong to our treatment group, $D = 1$. Defendants who were acquitted or their cases were dismissed according to the final ruling in their case belong to our comparison group, $D = 0$.

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Our outcome of interest, Y^*, is the "time-to-recidivism", i.e., the number of days it takes for a defendant to appear in court once again after the case's final ruling's date. Here, note that our outcome of interest is a duration variable and that some defendants may not recidivate by the end of our sampling period, though they may recidivate later. Putting it simply, we do not always observe Y^*, but rather observe a right-censored version of Y^*: $Y = min(Y^*, C)$, where C is a right-censoring variable. In our context, C is the follow-up period for each defendant, i.e., the number of days from their case's final ruling date to December 3rd, 2019.

Besides the censoring problem, it is important to be explicit about how we define recidivism. In this paper, a defendant i in a case j recidivated by the end of our sample if and only if defendant i's full name appears in a case $\bar{j}$ whose starting date is after case j's final sentence's date.[11] Then, we measure our outcome variable as the number of days between case j's final ruling's date and case $\bar{j}$'s starting date.[12] If defendant i did not recidivate by the end of the sampling period, then $Y = C$.

At this stage, it is important to stress that we are not adopting a more restrictive notion of "short-run" recidivism based on a fixed period, say two years, which could potentially allow us to "ignore" the censoring problem. Instead, we focus on time-to-recidivism directly, which, in our view, entails some important advantages. For instance, we do not need to pick a threshold to define (short-run) recidivism arbitrarily. Doing so may lead to potentially sensitive conclusions, as illustrated in an example in Appendix E.2.

However, if almost all defendants who recidivate do it in the short run, then focusing on short-run measures would be sufficient. But this is an empirical matter and should be handled as such. To assess if this is the case in our data, Figure 2 displays estimates of the right tail of the probability distribution function (PDF) of the uncensored potential outcome (Y^*) among cohorts defined based on the censoring variable. These descriptive results reveal that, in the case of the state of São Paulo, a non-negligible share of defendants have their first recidivism event in their fifth, sixth, or seventh year after their sentence's date, implying that analyzing long-term recidivism is practically relevant, which, in turn, requires us to tackle the censoring problem directly. See Appendix E.1 for additional motivations for leveraging time-to-recidivism as a key outcome of interest from a welfare maximizer decision-maker perspective.

As it will be clear in the next sections, our causal inference procedures leverage the availability of an instrumental variable Z with large support. In our context, the instrument Z is the trial judge's leniency rate. This variable equals the leave-one-out rate of punishment for

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[11]To match defendants' names across cases, we follow the same procedure as in Possebom (2023) and define a fuzzy match if the similarity between full names in two different cases is greater than or equal to 0.95 using the Jaro–Winkler similarity metric.

[12]Case $\bar{j}$ can be about any type of crime, including more severe crimes with a maximum sentence of over four years, while case j has to be about a crime whose maximum sentence is at most four years.

Figure 2: PDF of the Uncensored Outcome given the Defendant's Cohort
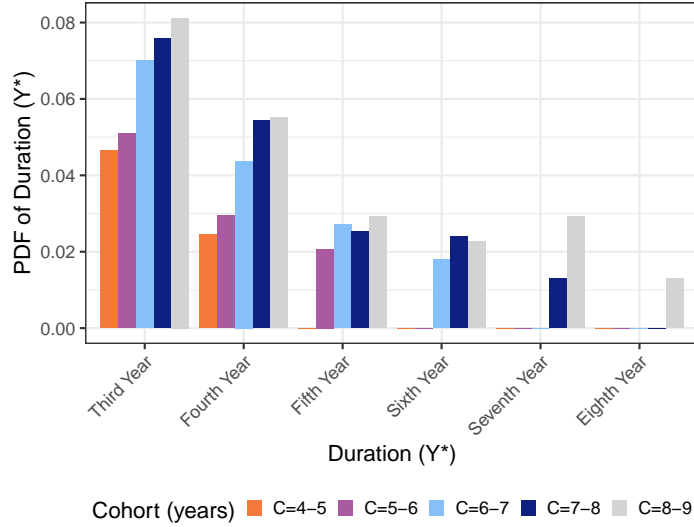
Notes: This figure shows the right tail of the probability mass function of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable, $\mathbb{P}\left[y_1 \leqslant Y^* \leqslant y_2 \mid C\right]$ where y_1 and y_2 define the bins indicated in the x-axis. In particular, these conditional PDFs are evaluated at six bins of the uncensored potential outcome. For example, "Third Year" denotes that the first recidivism event occurred between 730 days ($= y_1$) and 1095 days ($= y_2$), while "Fourth year" denotes that the first recidivism event occurred between 1095 days ($= y_1$) and 1460 days ($= y_2$). Each color denotes a different cohort: orange denotes defendants who are observed for at least four years and at most five years during our sampling period, purple denotes defendants who are observed for at least five years and at most six years, light blue denotes defendants who are observed for at least six years and at most seven years, dark blue denotes defendants who are observed for at least seven years and at most eight years, and gray denotes defendants who are observed for at least eight years and at most nine years. The y-axis denotes the value of the PDF.

each trial judge, where the defendant's own decision is excluded from this average.[13] We ensure that the minimum and maximum values of the Z are the same across both treatment arms to enforce better overlap properties.

Table 1 presents some summary statistics of our final sample. It shows the outcome's mean, 1st decile, 1st quartile and median for all defendants, for the defendants who were punished (treated group), and for the defendants who were not punished (comparison group). It also shows the sample size of each one of these three groups. The comparison between the treated and comparison groups suggests that being punished slightly harms defendants. However, this naive comparison ignores endogenous selection-into-treatment, right-censoring, and heterogeneous treatment effects. As such, one should be careful with such a comparison, as these descriptive statistics may not have a causal interpretation.

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[13] Similarly to Di Tella and Schargrodsky (2013) and Bhuller et al. (2020), we use the simple leave-one-out rate of punishment for each trial judge as our instrumental variable. Alternatively, we could have used the residualized leave-one-out rate of punishment as done by Agan et al. (2023), who remove court-district averages before computing each decision maker's rate. We choose to use the simple leave-one-out rate because we already include court-district fixed effects in our regression specifications, and each judge analyzes many cases as shown in Figure 1b.

Table 1: Descriptive Statistics — Outcome Variable

	Unconditional	Treated Group	Comparison Group
Mean	1,081	1,047	1,116
1st Decile	77	69	86
1st Quartile	364	321	430
Median	1082	1047	1127
Number of Observations	43,468	22,060	21,408

Note: The treated group receives a punishment, i.e., its defendants were fined or sentenced to community services because they were either convicted or signed a non-prosecution agreement. The comparison group did not receive a punishment, i.e., its defendants were acquitted or its cases were dismissed. The outcome variable measures the number of days between the case's final ruling's date and the first recidivism event if the defendant recidivates or the number of days between the case's final ruling's date and the end of the sampling period if the defendant did not recidivate. An observation is a case-defendant pair.

2.2 Causal questions of interest

For each defendant i, let $Y_i^*(1)$ be the potential time-to-recidivism if defendant i were punished with a fine or community service, and let $Y_i^*(0)$ be the potential time-to-recidivism if defendant i were not punished with a fine or community service. Defendant i's treatment effect is therefore $\theta_i = Y_i^*(1) - Y_i^*(0)$. Ideally, we would like to learn θ_i for all defendants. However, that is very challenging (if not impossible) when we allow for (a) heterogeneous treatment effects across defendants and (b) whether a defendant is punished or not being related to θ_i ("essential heterogeneity" as defined by Heckman et al. (2006)).

Due to these challenges, it is common for researchers to focus on aggregated summary measures of θ_i, such as the average treatment effect among "compliers" (Imbens and Angrist, 1994), defined as $LATE = \mathbb{E}\left[Y^*(1) - Y^*(0)|\text{Compliers}\right]$ (see, e.g., Agan et al., 2023; Bhuller et al., 2020; Huttunen et al., 2020).[14] Although interesting and policy-oriented, such aggregated measures of causal effects are unsuitable for highlighting important types of treatment effect heterogeneity. In particular, these parameters cannot answer whether defendants with high punishment resistance (i.e., defendants who would only be punished by very strict judges) would, on average, take longer to recidivate if they were punished. The same goes for defendants with lower punishment resistance. These are the exact types of causal questions that interest us in this paper. We want to go beyond LATE-type parameters and provide a more detailed picture of how alternative punishments heterogeneously affect recidivism with respect to the defendant's (unobserved) punishment resistance, which we denote by V. Here, punishment resistance may capture the evidence gathered against the defendant and additional defendant-specific characteristics.

One can measure the causal effect of fines and community services on time-to-recidivism for defendants with a given punishment resistance using the notion of distribution and quan-

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[14]As discussed before, these papers use a different outcome of interest Y^* that bypass the censoring issues we face. However, we can ignore these censoring issues while discussing our causal questions of interest (as this does not play a prominent role in it).

tile treatment effects.[15] All these causal parameters build on the Marginal Treatment Effects framework of Heckman and Vytlacil (2005) and can be used to answer complementary policy-relevant questions (Heckman and Vytlacil, 2005, Carneiro and Lee, 2009, Carneiro, Heckman and Vytlacil, 2011). We now carefully define and interpret them.

For treatment status $d \in \{0, 1\}$, let the distributional and quantile marginal treatment response functions be defined as

$$DMTR_d(y, v) := \mathbb{P}\left[Y^*(d) \leqslant y \mid V = v\right], \tag{2.1}$$

$$QMTR_d(\tau, v) := \inf\{y \in \mathbb{R}_+ : \mathbb{P}\left[Y^*(d) \leqslant y \mid V = v\right] \geqslant \tau\}, \tag{2.2}$$

where $y \in \mathbb{R}_+$, $\tau \in (0, 1)$ and $v \in [0, 1]$. All these counterfactual parameters have a clear interpretation. For instance, $DMTR_d(y, v)$ gives the proportion of defendants with punishment resistance v who would have already recidivated after y periods since the court's final ruling if they were treated ($d = 1$) or not ($d = 0$). Analogously, $QMTR_d(\tau, v)$ provide the τ-th quantile of the time-to-recidivism under treatment d, among defendants with punishment resistance v.

Based on these counterfactual objects, it is straightforward to define the Distributional and Quantile Marginal Treatment Effect functions:

$$DMTE(y, v) := DMTR_1(y, v) - DMTR_0(y, v), \tag{2.3}$$

$$QMTE(\tau, v) := QMTR_1(\tau, v) - QMTR_0(\tau, v). \tag{2.4}$$

Positive values of the QMTE function indicate that punishment by fines and community services increases the defendant's time-to-recidivism compared to no punishment (so treatment is working as intended). On the other hand, positive values of the DMTE function indicate that punishment by fines and community services increases the proportion of defendants who recidivate by time y compared to no punishment (so treatment is not working as intended). For policy effectiveness in our context, positive values of QMTE are "good", while negative values of DMTE are "good".

Remark 1. When analyzing the impact of judicial decisions on recidivism, many papers focus on distributional marginal treatment effects for a small set of values of y.[16] Here, we entertain the possibility of moving beyond small set of horizons by considering a continuous set of cutoffs y's or by focusing on different target parameters such as the quantile marginal treatment effects of the time-to-recidivism outcome. See Appendix E.2 for a simple illustration of the appeal of our approach compared to the traditional "small-set-of-horizons" approach.

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[15]In Appendix H, we also explore how to define and identify types of average treatment effects. Importantly, censoring causes technical issues when defining parameters based on average effects, and we propose two alternative solutions that overcome those challenges.

[16]See, e.g., Agan et al. (2023), Bhuller et al. (2020), Giles (2023), Huttunen et al. (2020), Klaassen (2021), and Possebom (2023).

3   Econometric framework and identification results

We face some challenges in identifying the causal parameters of interest described in the previous section. As usual, potential outcomes are only (potentially) observed under one treatment status, i.e., $Y_i^* = Y_i^*(1) \cdot D_i + Y_i^*(0) \cdot (1 - D_i)$. Furthermore, our outcome of interest is subject to right-censoring, implying that we do not always observe Y^* but rather observe $Y_i = \min\{Y_i^*, C_i\}$, where C is the censoring variable. In the case of draws, we assume that Y_i^* happens before C_i, as is customary in survival analysis. Finally, we also expect that treatment statuses are related to the potential outcomes and possibly related to the censoring variable.

To tackle all these issues, we build on the MTE framework of Heckman and Vytlacil (2005) and extend it to handle duration outcomes. Toward this end, we consider a threshold-crossing treatment selection model

$$D = \mathbf{1}\left\{P\left(Z, C\right) \geqslant V\right\}, \tag{3.1}$$

where Z is an observed instrumental variable (with support $\mathcal{Z} \subset \mathbb{R}$), C is an observed censoring variable (with support $\mathcal{C} \subset \mathbb{R}_+$), and V is a latent heterogeneity variable that captures the unobserved treatment resistance (with support $(0, 1)$). The function $P : \mathcal{Z} \times \mathcal{C} \to \mathcal{P} \subseteq [0, 1]$ is unknown and captures the willingness to take the treatment for each value of Z and C. Importantly, our treatment selection model (3.1) imposes monotonicity (Imbens and Angrist, 1994; Vytlacil, 2002).

To better understand Equation (3.1), let us go back to our empirical context and explain each component of it. Our instrumental variable Z is a measure of the trial judge's leniency, which arguably does not affect time-to-recidivism other than through the judge's decision to punish or not. Our censoring variable C captures the time between the defendant's sentence date and the end of our sampling period, and it is observed for all defendants. C can also capture seasonality patterns, as it is fully determined depending on the sentence's date. The function P captures the trial judge's punishment criteria, and it allows trial judges to update their punishment criteria over time (Bhuller and Sigstad, 2024), as it includes C as an argument. Finally, the variable V can be interpreted as unobserved punishment resistance, and it captures, among other things, the amount of criminal evidence in the defendant's favor. The higher the V, the less likely the defendant will be punished, all else equal. As already discussed, Y^* captures the length of time between the defendant's sentence date and her next criminal case's starting date, and Y is the minimum of Y^* and time from the sentence's date to the end of our sampling period, C.

3.1 Assumptions

In our setup, the available data for the researcher are $\{Y_i, C_i, D_i, Z_i\}_{i=1}^n$, while $Y_i^*(0)$, $Y_i^*(1)$, Y_i^* and V_i are latent variables. Henceforth, we assume that $\{Y_i, C_i, D_i, Z_i\}_{i=1}^n$ are independently and identically distributed as (Y, C, D, Z). For simplicity, we drop exogenous covariates from the model and focus on the case with a single instrument. All results derived in the paper hold conditionally on covariates and can be extended to the case with multiple instruments. Since we deal with a time-to-event outcome, Y is non-negative by construction.

In what follows, we present a set of five assumptions (Assumptions 1-5) that will allow us to point-identify the DMTE and the QMTE functions across a range of thresholds and quantile points. These assumptions are related to those imposed by Heckman and Vytlacil (2005) and Frandsen (2015) and involve assuming that censoring is not related to the potential outcomes $Y^*(d)$.

We now state our five assumptions and contextualize each of them to our empirical setup.

Assumption 1 (Random Assignment). *Conditional on C, the potential outcomes $Y^*(0)$, $Y^*(1)$ and V are independent of the instrument Z, i.e.,*

$$Z \perp\!\!\!\perp (Y^*(0), Y^*(1), V) | C.$$

Assumption 1 is an exogeneity assumption and is common in the literature about instrumental variables with censored outcomes (Frandsen, 2015). In our empirical application, this assumption holds conditional on the court district because trial judges are randomly assigned to cases within each court district. Importantly, judges are assigned to criminal cases based on a computer algorithm that creates a lottery of judges and there is no suspicion in the press that this software is manipulated.

Note also that Assumption 1 allows the instrument to depend on the censoring variable. In our empirical application, this flexibility is useful because the trial judge's punishment rate may depend on the case's sentence date if judges who entered the Judiciary more recently are more lenient than judges who retired at the beginning of our sampling period, for example.

Assumption 2 (Propensity Score is Continuous). *Conditional on C, $P(z, c)$ is a nontrivial function of z and the random variable $P(Z, c) | C = c$ is absolutely continuous in Z, with support given by an interval $\mathcal{P} := [\underline{p}, \overline{p}] \subseteq [0, 1]$ for any $c \in \mathcal{C}$.[17]*

Assumption 2 is a rank condition, intuitively imposing that the instrument is locally relevant. In addition, we implicitly assume that the support of the propensity score does not vary with the value of C. In our application, this implicit assumption is plausible because the judges

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[17]The assumption that $\mathcal{P}$ is an interval is made for notational simplicity. All the proofs can be easily extended to the case where $\mathcal{P}$ is a set with a non-empty interior.

are mostly the same over time. Furthermore, the judge's lenience rate has a good amount of variation since we observe 525 judges who use many different punishment criteria (Figure 4).[18]

Assumption 3 (V is continuous). *The distribution of the latent heterogeneity variable V conditional on C is absolutely continuous with respect to the Lebesgue measure.*

Assumption 3 is a regularity condition that allows us to normalize the marginal distribution of $V|C$ to be the standard uniform. Consequently, we can write $P(z,c) = \mathbb{P}[D = 1 | Z = z, C = c]$ for any $z \in \mathcal{Z}$ and $c \in \mathcal{C}$.

Assumption 4 (Overlap). *Conditional on C, all treatment groups exist, i.e., $\mathbb{P}[D = d | C = c] \in (0,1)$ for any $d \in \{0,1\}$ and any $c \in \mathcal{C}$.*

Assumption 4 is a regularity condition about overlap.

Assumption 5 (Random Censoring). *The censoring variables are independent of the uncensored potential outcomes given the latent heterogeneity V, i.e.,*

$$C \perp\!\!\!\perp (Y^*(0), Y^*(1)) | V.$$

Assumption 5 is an exogeneity assumption and is common in the literature about duration outcomes (Frandsen, 2015; Sant'Anna, 2016; Delgado et al., 2022). When combined with Assumption 3, Assumption 5 implies that C is unconditionally independent of the uncensored potential outcomes, i.e., $C \perp\!\!\!\perp (Y^*(0), Y^*(1))$. In our empirical application, this restriction imposes that the case's sentence date is independent of the defendant's decision to commit another crime in the future, i.e., potential recidivism is stationary.

Importantly, Assumption 5 imposes that controlling for V accounts for all sources of endogeneity coming through the censoring variable. This assumption can be restrictive since endogeneity might still be present once controlling for the latent heterogeneity. If the researcher believes that this assumption is too strong in a particular application, she can use alternative assumptions that are sufficient to partially identify the distributional marginal treatment effect and some quantile marginal treatment effects when the outcome variable is right-censored. We discuss two alternative partial identification strategies in Appendix G.

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[18] Since our main identification result is fully nonparametric, Assumption 2 requires sufficient (continuous) variation in the instrument, which induces continuous variation in the propensity score, which generates the variation we need on the outcomes of interest to identify the effects for every value of V, the latent heterogeneity. Nevertheless, our proposed estimation procedure is semi-parametric and relies on series approximations for the propensity score. If one treats the series as fixed (meaning that the first stage is parametric), our approach allows for discrete instruments (Brinch et al., 2017). The extrapolation and interpolation induced by the parametric second stage permit Z (and P) to vary discretely while still being used to identify the effects of interest.

3.2 Identification

We present our point-identification results that rely on Assumptions 1-5.

First, define

$$\gamma_d(y, v, c) = \frac{d}{dv} \mathbb{P}\left[Y \leqslant y, D = d \mid P(Z, C) = v, C = c\right].$$

We now state our main identification result: point-identification of the $DMTR$ functions.

Proposition 3.1. *Suppose that Assumptions 1-5 hold. Then, for any $d \in \{0, 1\}$, $y < \gamma_C$ and $v \in \mathcal{P}$,*

$$DMTR_d(y, v) = (2d - 1) \cdot \mathbb{E}\left[\gamma_d(y, p, C) \mid P(Z, C) = v, C > y\right],$$

where $\gamma_C := \inf\left\{c \in \overline{\mathbb{R}} : \mathbb{P}[C \leqslant c] = 1\right\}$ is the upper bound of the support of the censoring variable C.

Proof. See Appendix A.2. ∎

The above proposition shows how we can point-identify the distributional marginal treatment response for a given unobserved treatment resistance v. It involves first taking the derivative of the conditional joint distribution of the realized outcome Y and treatment status D given the propensity score $P = v$ and the censoring variable being above y ($C = y + \delta$ for $\delta > 0$) with respect to v, and then integrating over all values $\delta > 0$ such that the $y + \delta$ remains in the support of the censoring variable C. Differently from the results in Carneiro and Lee (2009) and Carneiro et al. (2011), we need to tackle the right-censoring problem, which manifests in our results by having to condition on $C = y + \delta$, so that $C > y$, and then integrating over δ.

Furthermore, our results are specific to the $DMTR_d(y, v)$ function, and not for a generic transformation of $Y^*(d)$, say $G(Y^*(d))$ as in Carneiro and Lee (2009). This follows from the fact that we may not be able to identify the $DMTR_d(y, v)$ over all values of y in the support of $Y^*(d)$, as a consequence of the censoring problem. Having said that, there are several functions that we can actually nonparametrically point-identify without additional restrictions and under standard regularity conditions, including the QMTE functions for a range of quantiles. We state these results, which use only Assumptions 1-5 and a regularity condition, as a corollary for convenience. The proof is a direct consequence of the previous proposition and the definition of quantiles.

Corollary 3.1. *Suppose that Assumptions 1-5 and Assumption B.7 listed in the Appendix B hold. Then the $QMTE(\tau, v)$ function (Equation (2.4)) is point-identified for any $v \in \mathcal{P}$ and $\tau \in (0, \overline{\tau}(v))$, where $\overline{\tau}(v) := \min\{\overline{\tau}_0(v), \overline{\tau}_1(v)\}$ and $\overline{\tau}_d(v) := DMTR_d(\gamma_C, v)$ for any $d \in \{0, 1\}$.*

Notice that the right-tail of the $DMTR_d(\cdot, v)$ may be differentially affected by the censoring problem, implying that $\overline{\tau}_1(v)$ may be different from $\overline{\tau}_0(v)$. As a consequence, we can only

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identify the $QMTE\,(\tau,v)$ over the common range of identified quantiles among treated and untreated units.

4   Estimation and inference

In this section, we provide algorithms on how to semiparametrically estimate the DMTE and QMTE functions based on the identification results described in Proposition 3.1 and Corollary 3.1. We discuss two sets of results in this section. First, we present generic algorithms to estimate the marginal treatment effect functionals that remain agnostic about the type of estimators used to estimate the nuisance functions (Section 4.1). These results are useful for pinpointing intuition and providing templates for flexible estimation procedures. However, pinning down the asymptotic properties of such estimators at this level of generality is rather challenging, especially regarding inference. To ameliorate this, we provide practical and formally justified estimation and inference procedures based on a more restricted class of estimators for the nuisance functions (Section 4.2).

4.1   Generic estimation procedure

We first present a generic algorithm one can use to estimate DMTE functions across a grid of threshold points $\{y_k\}_{k=0}^K$. The algorithm will make use of an estimator for the propensity score, $P(Z,C) = \mathbb{E}\,[D|Z,C]$, and the conditional distribution of $Y \cdot \mathbf{1}\,\{D=d\}$ given $P\,(Z,C)$ and C for any $d \in \{0,1\}$. This algorithm builds on Proposition 3.1. Recall that our data consist of $i.i.d.$ observations $\{Y_i, C_i, D_i, Z_i\}_{i=1}^n$, where n is the sample size.

Algorithm 4.1 (Generic Estimation of DMTE function)**.**

1. *Semiparametrically (or nonparametrically) estimate the propensity score $P \colon \mathcal{Z} \times \mathcal{C} \to [0,1]$. Denote the fitted propensity score values by $\widehat{P}_i$.*

2. *Define a grid of values for the duration outcome Y, $\{y_k\}_{k=0}^K$, such that $y_k > y_{k-1}$ for any $k \in \{1,\dots,K\}$ and $K \in \mathbb{N}$.*

3. *For each $k \in \{0,\dots,K\}$ and each $d \in \{0,1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\,\{D=d\}$ given $P\,(Z,C)$, and C, that is,*
$$\Gamma(P, C; y_k, d) = \mathbb{E}\,[\mathbf{1}\,\{Y \leqslant y_k, D = d\}\,|P, C]\,.$$
   Since the propensity score for unit i, P_i, is unknown, use the estimated fitted values from Step 1. Denote the estimated fitted values by $\widehat{\Gamma}(\widehat{P}_i, C_i; y_k, d) = \widehat{\Gamma}_{d,k,i}$.

4. *For each $k \in \{0,\dots,K\}$ and $d \in \{0,1\}$, estimate the derivative of $\Gamma(P, C; y_k, d)$ with respect to P. Since $\Gamma(P, C; y_k, d)$ is unknown, use the estimated $\widehat{\Gamma}_{d,k,i}$. Denote the estimated derivative evaluated at $P = v, C = c$ by $\widehat{\gamma}_d(y_k, v, c)$, where $v \in \mathcal{P}$, and $c \in \mathcal{C}$.*

5. *For each $k \in \{0, \dots, K\}$ and each $d \in \{0, 1\}$, estimate $DMTR_d(y_k, v)$ by averaging $(2d - 1)\widehat{\gamma}_d(y_k, v, c)$ over values of c such that $c > y_k$,*

$$\widehat{DMTR}_d(y_k, v) = (2d - 1)\widehat{\mathbb{E}}\left[\widehat{\gamma}_d(y_k, v, C) \mid C > y_k, \widehat{P} = v\right],$$

where $\widehat{\mathbb{E}}[\cdot|\cdot]$ is a generic estimator for a conditional expectation.

6. *For each value $v \in \mathcal{P}$ and $d \in \{0, 1\}$, ensure that $\widehat{DMTR}_d(y_k, v)$ is non-decreasing in y_k, and bounded between zero and one.*

7. *For each $k \in \{0, \dots, K\}$, estimate $DMTE(y_k, v)$ using*

$$\widehat{DMTE}(y_k, v) := \widehat{DMTR}_1(y_k, v) - \widehat{DMTR}_0(y_k, v).$$

Building on Algorithm 4.1, it is straightforward to compute the QMTE functions. More specifically, from Step 6, we have that for both $d = 1$ and $d = 0$, $DMTR_d(y, v)$ is monotone in y for a given v, as any cumulative distribution function should be. Thus, one can invert these to compute the quantile marginal treatment response functions, and then take their differences to compute the QMTE function. More precisely, for each $d \in \{0, 1\}$ and a quantile τ, a generic estimator of the $QMTR_d(\tau, v)$ is given by

$$\widehat{QMTR}_d(\tau, v) := \min_{k \in \{0, \dots, K\}}\left\{y_k : \widehat{DMTR}_d(y_k, v) \geqslant \tau\right\}.$$

The QMTE estimator for any $\tau \in [0, \overline{\tau}(v))$ is given by

$$\widehat{QMTR}(\tau, v) = \widehat{QMTR}_1(\tau, v) - \widehat{QMTR}_0(\tau, v).$$

Remark 2. The above procedures do not explicitly account for covariates X. However, every single step of Algorithm 4.1 can be thought as implicitly conditioning on covariate values $X = x$. In such cases, one would get a conditional version of the DMTE function, and, consequentially, a conditional version of the QMTE functions. If one were interested in the "unconditional" version of these functionals, all one needs to do is integrate the DMTE functional over values of the distribution of X given the propensity score $P = v$, and then follow the same steps as described in the paragraphs after Algorithm 4.1. If covariates are of moderate dimensions or data for each covariate stratum is not-so-large to justify asymptotic approximations, one may be interested in imposing additional restrictions to bypass the "curse-of-dimensionality". We follow this path in the next section.

4.2 Semiparametric estimation and inference procedures

This section provides a more specific procedure to semiparametrically estimate the marginal treatment effect functionals discussed before. The steps we follow are similar to those in Algorithm 4.1, but we are more specific about choosing estimators for the nuisance functions. This degree of specificity allows us to establish large-sample statistical guarantees and provide asymptotically valid inference procedures for the target functionals of interest.

Here, we also allow for additional covariates X to enter into the model, so we have data on $\{Y_i, C_i, D_i, Z_i, X_i'\}_{i=1}^n$. In the context of our application about the effect of alternative punishments to misdemeanor offenses on time-to-recidivism in Brazil, X is a set of court district indicators. For this reason, we focus on the case where all X's are discrete.

We start discussing how we estimate the propensity score. Similar to Carneiro and Lee (2009), we model $P(Z, C, X) := \mathbb{E}[D|Z, C, X]$ using an additive partially linear series regression

$$P(Z, C, X) = \alpha_0 + X'\alpha_X + C\alpha_C + \varphi(Z), \tag{4.1}$$

where $(\alpha_0, \alpha_X', \alpha_C)'$ are unknown finite-dimensional parameters, and φ is an unknown (infinite-dimension) function. In our context, the partially linear additive specification (Eqaution (4.1)) allows one to pool information from different court districts and run a single propensity score model for all courts. Alternatively, one could use a different propensity score model for each district, at the cost of getting arguably much less precise estimates.[19]

In practice, one would approximate $\varphi(\cdot)$ using a linear combination of the vector of basis functions $\psi^L(z) = (\psi_1(z), \psi_2(z), \ldots, \psi_L(z))'$, for $L \in \mathbb{N}$. That is, $\varphi(z) \approx \psi^L(z)'\alpha_Z$, such that, as $L \to \infty$, the approximation error shrinks to zero. For simplicity, we pick a polynomial basis function, $\psi^L(z) = (z, z^2, \ldots, z^L)'$, though other options such as B-splines are also possible (Chen, 2007). Note that all the series coefficients can be estimated via ordinary least squares, i.e.,

$$\hat{\theta}^{fs} = \underset{\theta^{fs} \in \Theta^{fs}}{\arg\min} \; n^{-1} \sum_{i=1}^n \left(D_i - \alpha_0 - X_i'\alpha_X - C_i\alpha_C - \psi^L(Z_i)'\alpha_Z,\right)^2 \tag{4.2}$$

where $\hat{\theta}^{fs} = (\hat{\alpha}_0, \hat{\alpha}_X', \hat{\alpha}_C, \hat{\alpha}_Z))'$. Thus, we can compute the fitted propensity score values

$$\widetilde{P}_i = \widetilde{P}(Z_i, C_i, X_i) = \hat{\alpha}_0 + X_i'\hat{\alpha}_X + C_i\hat{\alpha}_C + \psi^L(Z_i)'\hat{\alpha}_Z. \tag{4.3}$$

In finite samples, $\widetilde{P}_i$ might be negative or larger than one. To handle this, we follow Carneiro and Lee (2009) and use the trimmed version of $\widetilde{P}_i$ as our estimator,

$$\widehat{P}_i = \widetilde{P}_i + (1 - \epsilon - \widetilde{P}_i) \cdot \mathbf{1}\{\widetilde{P}_i > 1\} + (\epsilon - \widetilde{P}_i) \cdot \mathbf{1}\{\widetilde{P}_i < 0\}, \tag{4.4}$$

for a sufficiently small ϵ. Our application uses $\epsilon = 0.01$, though this is not material as only one observation is trimmed.

Next, we move into the estimation of the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given P, C, X for $d \in \{0, 1\}$. Here, we follow the distribution regression approach introduced by Foresi and Peracchi (1995) and further formalized by Chernozhukov, Fernández-Val and Melly (2013). The idea is to pose a model with "varying coefficients" for the conditional distribution of $Y \cdot \mathbf{1}\{D = d\}$, $d \in \{0, 1\}$,

$$\Gamma(P, C, X; y, d) \equiv \mathbb{E}[\mathbf{1}\{Y \leq y, D = d\}|P, C, X]$$

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[19]Alternatively, one could use a semiparametric logit model $P(Z, C, X) = \exp\{\alpha_0 + X'\alpha_X + C\alpha_C + \varphi(Z)\}/(1 + \exp\{\alpha_0 + X'\alpha_X + C\alpha_C + \varphi(Z)\})$. In the Appendix, we show that our regularity conditions could still be satisfied. The same is true for a fully nonparametric series estimator.

$$= \Lambda\left(\beta_0\left(y, d\right) + X'\beta_X(y, d) + C\beta_C(y, d) + P\beta_P(y, d)\right) \text{ a.s.} \tag{4.5}$$

where $\theta_0(\cdot, \cdot) = (\beta_0\left(\cdot, \cdot\right), \beta_X\left(\cdot, \cdot\right)', \beta_C\left(\cdot, \cdot\right), \beta_P\left(\cdot, \cdot\right))' \mapsto \Theta \subseteq \mathbb{R}^{3+k_X}$ is a vector of nonparametric functions, k_X is the dimension of X, and Λ is a known link function.[20] For concreteness, we focus on a logistic link function, $\Lambda(\cdot) = \exp(\cdot)/(1 + \exp(\cdot))$.

To estimate these unknown functions, we first need to acknowledge that the propensity score P_i is not observed. However, we can use the "generated regressor" $\widehat{P}_i$ from Equation (4.4). Once we replace P_i with $\widehat{P}_i$, we can then leverage the insights of Foresi and Peracchi (1995) and Chernozhukov et al. (2013) by noticing that, for a fixed y and d, Equation (4.5) is a binary regression problem. Consequently, we can pointwise estimate these parameters by maximizing the (feasible) conditional likelihood function

$$\hat{Q}(\theta; y, d) = \frac{1}{n}\sum_{i=1}^n \ln \ell_\theta(\mathbf{1}\{Y_i \leqslant y, D_i = d\}, X_i, C_i, \widehat{P}_i; y, d) \tag{4.6}$$

with

$$\ell_\theta(b, x, c, p; y, d) = \Lambda\left(w'\theta\right)^b \left(1 - \Lambda\left(w'\theta\right)\right)^{1-b},$$

and $w = (1, x', c, p)'$. Thus, the distribution regression (DR) estimator of $\theta_0(y, d)$ is given by

$$\hat{\theta}\left(y, d\right) = \arg\max_{\theta \in \Theta} \hat{Q}_n\left(\theta; y, d\right). \tag{4.7}$$

Notice that computing the distribution regression estimators for several (y, d) points only requires running a sequence of binary regressions. This can be performed in any statistical software.

Next, note that the derivative of Γ (Equation (4.5)) with respect to P can computed in closed-form for each (y, d),

$$\gamma_d(y, v, c, x) \quad := \quad \frac{d}{dv}\Gamma(v, c, x; y, d) = \beta_P(y, d)\ \Gamma(v, c, x; y, d)(1 - \Gamma(v, c, x; y, d)), \tag{4.8}$$

where we explored that Λ is the logistic link function. Denote the distribution regression estimated fitted values of $\gamma_d(y, v, c, x)$ by

$$\widehat{\gamma}_d(y, v, c, x) = \widehat{\beta}_P(y, d)\ \widehat{\Gamma}(v, c, x; y, d)(1 - \widehat{\Gamma}(v, c, x; y, d)), \tag{4.9}$$

where the distribution regression coefficients are as in Equation (4.7), and

$$\widehat{\Gamma}(v, c, x; y, d) = \frac{\exp\left(\widehat{\beta}_0\left(y, d\right) + x'\widehat{\beta}_X(y, d) + c\widehat{\beta}_C(y, d) + v\widehat{\beta}_P(y, d)\right)}{1 + \exp\left(\widehat{\beta}_0\left(y, d\right) + x'\widehat{\beta}_X(y, d) + c\widehat{\beta}_C(y, d) + v\widehat{\beta}_P(y, d)\right)}.$$

Using Equation (4.9), we can estimate

$$DMTR_d(y, v, x) := \mathbb{P}\left[Y^*(d) \leqslant y \,|\, V = v, X = x\right].$$

To do so, let $n_{d,x,y} = \sum_{i=1}^n \mathbf{1}\{D_i = d, X_i = x, C_i > y\}$ denote the sample size with treat-

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[20]This class of distribution regression models nests and extends many traditional duration models such as the proportional hazard model (Cox, 1972) and the accelerated time model (Kalbfleisch and Prentice, 1980). See Delgado et al. (2022) for a discussion.

ment status d, covariate value x, and censoring variable above y. Our proposed estimator for $DMTR_d(y, v, x)$ is given by

$$\widehat{DMTR}_d(y, v, x) = (2d - 1)\frac{\sum_{i=1}^{n} \mathbf{1}\{D_i = d, X_i = x, C_i > y\}\, \hat{\gamma}_d(y, v, C_i, x)}{n_{d,x,y}}. \quad (4.10)$$

Since $\widehat{DMTR}_d(y, v, x)$ is an estimator for a conditional distribution, it needs to be non-decreasing in y for all $(d, v, x) \in \{0, 1\} \times \mathcal{P} \times \mathcal{X}$. However, this may not be the case in finite samples. We recommend using the rearrangement procedure of Chernozhukov, Fernandez-Val and Galichon (2009).

Based on Equation (4.10), we can then estimate $DMTE(y, v, x) \coloneqq DMTR_1(y, v, x) - DMTR_0(y, v, x)$ using

$$\widehat{DMTE}(y, v, x) \coloneqq \widehat{DMTR}_1(y, v, x) - \widehat{DMTR}_0(y, v, x). \quad (4.11)$$

Analogously, one can estimate $QMTE(\tau, v, x)$ functionals using

$$\widehat{QMTE}(\tau, v, x) = \widehat{QMTR}_1(\tau, v, x) - \widehat{QMTR}_0(\tau, v, x), \quad (4.12)$$

where $\widehat{QMTR}_d(\tau, v, x) = \inf\{y \in \mathbb{R}_+ : \widehat{DMTR}_d(y, v, x) \geqslant \tau\}$.

We summarize all these estimation steps in the following algorithm.

Algorithm 4.2 (Semiparametric Estimation of DMTE and QMTE functionals).

1. *Semiparametrically estimate the propensity score using the series partially linear model in Equation (4.1). Denote its trimmed fitted propensity score values by $\hat{P}_i$ as defined in Equation (4.4).*

2. *Define a grid of values for the duration outcome Y: $\{y_k\}_{k=0}^{K}$ such that $y_k > y_{k-1}$ for any $k \in \{1, \ldots, K\}$ and $K \in \mathbb{N}$.*

3. *For each $k \in \{0, \ldots, K\}$ and each $d \in \{0, 1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z, C)$, C, and X using the distribution regression model in Equation (4.5), with estimated coefficients in Equation (4.7).*

4. *For each $k \in \{0, \ldots, K\}$ and $d \in \{0, 1\}$, estimate the derivative of the distribution regression model with respect to P as in Equation (4.8). Denote its estimated fitted value for a given x by $\hat{\gamma}_d(y_k, v, c, x)$ as in Equation (4.9).*

5. *For each $k \in \{0, \ldots, K\}$ and each $d \in \{0, 1\}$, compute $\widehat{DMTR}_d(y_k, v, x)$ as in Equation (4.10).*

6. *For each value $v \in \mathcal{P}$, $d \in \{0, 1\}$, and for a given x, ensure that $\widehat{DMTR}_d(y_k, v, x)$ is non-decreasing in y_k, and bounded between zero and one.*

7. *For each $k \in \{0, \ldots, K\}$, estimate the $DMTE(y_k, v, x)$ using Equation (4.11).*

8. *Estimate the $QMTE(\tau, v, x)$ using Equation (4.12), with*
$$\widehat{QMTR}_d(\tau, v, x) = \min\{y \in \{y_k\}_{k=0}^{K} : \widehat{DMTR}_d(y, v, x) \geqslant \tau\}, \ d \in \{0, 1\}.$$

The next theorem establishes the large-sample properties of our proposed estimators. We defer all the regularity conditions to the appendix to streamline the presentation. Let $\overline{\tau}(v, x) :=$ $\min\{\overline{\tau}_0(v, x), \overline{\tau}_1(v, x)\}$ and $\overline{\tau}_d(v, x) := DMTR_d(\gamma_C, v, x)$ for any $d \in \{0, 1\}$.

Theorem 4.1. *Suppose that Assumptions 1-5 and Assumptions B.1-B.7 listed in Appendix B hold. Then, as $n \to \infty$,*

(a) for each fixed $y < \gamma_C$, $v \in \mathcal{P}$, and $x \in \mathcal{X}$,

$$\sqrt{n}\left(\widehat{DMTE}(y, v, x) - DMTE(y, v, x)\right) \xrightarrow{d} N(0, V_{y,v,x}^{dmte}),$$

with $\widehat{DMTE}(y, v, x)$ as defined in Equation (4.11) and $V_{y,v,x}^{dmte}$ as defined in Appendix B.

(b) for each fixed $\tau \in (0, \overline{\tau}(v, x))$, $v \in \mathcal{P}$, and $x \in \mathcal{X}$,

$$\sqrt{n}\left(\widehat{QMTE}(\tau, v, x) - QMTE(\tau, v, x)\right) \xrightarrow{d} N(0, V_{\tau,v,x}^{qmte}),$$

with $\widehat{QMTE}(\tau, v, x)$ as defined in Equation (4.12) and $V_{\tau,v,x}^{qmte}$ as defined in Appendix B.

Theorem 4.1 follows from first deriving the influence function of the $DMTR$ functions (Equation (4.10)), paying particular attention to quantifying the estimation effect arising from replacing the true propensity score with the estimated one. After this step, all the results follow from the functional delta method and the continuous mapping theorem. The proof strategy is similar to Rothe (2009).

Although Theorem 4.1 indicates that one can potentially conduct inference using plug-in estimates of the variance, this procedure would involve estimating additional nuisance functions and could be cumbersome in practice. To avoid this issue, we propose using a weighted bootstrap procedure as in Ma and Kosorok (2005) and Chen and Pouzo (2009). This bootstrap procedure is very straightforward to implement, as described in the next algorithm.

Algorithm 4.3 (Weighted-Bootstrap Implementation)**.**

1. *Estimate DMTE and QMTE according to Algorithm 4.2.*

2. *Generate $\{\omega_i, i = 1, \ldots, n\}$ as a sequence of independent and identically distributed non-negative random variables with mean one, variance one, and finite third moment (e.g., $\omega_i \sim Exp(1)$).*

3. *Compute the propensity score coefficients associated with Equation (4.1) by minimizing the weighted least squares function, i.e,*

$$\hat{\theta}^{fs,*} = \underset{\theta^{fs} \in \Theta^{fs}}{\arg\min}\ n^{-1}\sum_{i=1}^{n}\omega_i\left(D_i - \alpha_0 - X_i'\alpha_X - C_i\alpha_C - \psi^L(Z_i)'\alpha_Z,\right)^2 \tag{4.13}$$

where $\hat{\theta}^{fs,*} = (\hat{\alpha}_0^*, \hat{\alpha}_X^{*,\prime}, \hat{\alpha}_C^*, \hat{\alpha}_Z^*))'$. Denote its trimmed fitted propensity score values by $\hat{P}_i^*$ as defined in Equation (4.4), but with $\hat{\theta}^{fs,*}$ in place of $\hat{\theta}^{fs}$.

4. *Consider the same grid of values for the duration outcome Y as defined in Step 2 of Algorithm 4.2.*

5. *For each $k \in \{0, \ldots, K\}$ and each $d \in \{0, 1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z, C)$, C, and X using the distribution regression model (Equation (4.5)) with estimated coefficients*

$$\hat{\theta}^*(y, d) = \arg\max_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \omega_i \ln \ell_\theta(\mathbf{1}\{Y_i \leqslant y, D_i = d\}, X_i, C_i, \hat{P}_i^*; y, d). \qquad (4.14)$$

6. *Follow Steps 4-9 of Algorithm 4.2 using $\hat{\theta}^*(y, d)$ instead of $\hat{\theta}(y, d)$. Denote by $\widehat{DMTE}^*(y_k, v, x)$ and $\widehat{QMTE}^*(\tau, v, x)$ the distributional and quantile marginal treatment effects estimates.*

7. *Repeat Steps 2-6 B times, e.g., $B = 399$, and collect $\left\{ \left(\widehat{DMTE}^*(y_k, v, x) \right)_b, b = 1 \ldots, B \right\}$. Do the same for the $\widehat{QMTE}^*(\tau, v, x)$.*

8. *Obtain the $(1 - \alpha)$ quantile of $\left\{ \left| \left(\widehat{DMTE}^*(y_k, v, x) - \widehat{DMTE}(y_k, v, x) \right)_b \right|, b = 1 \ldots, B \right\}$, $c^{dmte,*}(y_k, v, x; \alpha)$. Compute the analogous critical values based on $\widehat{QMTE}^*(\tau, v, x)$.*

9. *Construct the $1 - \alpha$ (pointwise) confidence interval for $DMTE(y_k, v, x)$ as $\hat{C}^{dmte}(y_k, v, x) = [\widehat{DMTE}(y_k, v, x) \pm c^{dmte,*}(y_k, v, x; \alpha)]$. Define $\hat{C}^{qmte}(\tau, v, x; \alpha)$ analogously.[21]*

The next theorem establishes that our weighted bootstrap procedure has asymptotically correct coverage.

Theorem 4.2. *Under the assumptions of Theorem 4.1, for any $0 < \alpha < 1$, and for each $v \in \mathcal{P}$, $x \in \mathcal{X}$, $y < \gamma_C$, and $\tau \in (0, \overline{\tau}(v, x))$, for $n \to \infty$,*

(a) $\mathbb{P}\left(DMTE(y_k, v, x) \in \hat{C}^{dmte}(y_k, v, x; \alpha) \right) \to 1 - \alpha$,

(b) $\mathbb{P}\left(QMTE(\tau, v, x) \in \hat{C}^{qmte}(\tau, v, x; \alpha) \right) \to 1 - \alpha$.

Note that all functionals in Theorems 4.1 and 4.2 provide a covariate-specific treatment effect. In our application's context, we can get court-district-specific DMTE and QMTE estimates of the effect of fines and community service sentences on time-to-recidivism. An advantage of this approach is that one can better understand treatment effect heterogeneity across districts. However, with 193 court districts in the State of São Paulo, it may be desirable to further aggregate the MTE functionals as a way to summarize the obtained effects.

There are several potential aggregations one could entertain. In our specific context, we decided to aggregate the court-district-specific DMTE functionals across court districts using the proportion of cases per court district as weights. Let $w_x = \mathbb{P}(X = x)$ denote the probability of a covariate X takes the value x, which, in our case, denotes the true proportion of cases assigned to a court district x. Let $\hat{w}_x = n^{-1} \sum_{i=1}^{n} \mathbf{1}\{X_i = x\}$ be the plugin estimator of w_x.

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[21] If the researcher wants to compute uniform confidence bands instead of pointwise confidence intervals, it is straightforward to adapt the sup-t procedure proposed by Olea and Plagborg-Moller (2018) to our setting.

For each $d \in \{0, 1\}$, $y \in \mathcal{Y}$ and $v \in \mathcal{P}$, let

$$DMTR_d^{avg}(y, v) = \mathbb{E}\left[DMTR_d(y, v, X)\right] = \sum_{x \in \mathcal{X}} w_x \, DMTR_d(y, v, x).$$

Analogously, let $DMTE^{avg}(y, v) = DMTR_1^{avg}(y, v) - DMTR_0^{avg}(y, v)$ and $QMTE^{avg}(\tau, v) = QMTR_1^{avg}(\tau, v) - QMTR_0^{avg}(\tau, v)$, where $QMTR_d^{avg}(\tau, v) := \inf\{y \in \mathbb{R}_+ : DMTR_d^{avg}(y, v) \geqslant \tau\}$. All these functionals can be straightforwardly estimated using functionals of

$$\widehat{DMTR}_d^{avg}(y, v) = \sum_{x \in \mathcal{X}} \widehat{w}_x \, \widehat{DMTR}_d(y, v, x),$$

with $\widehat{DMTR}_d(y, v, x)$ as in Equation (4.10), just like in Equations (4.11)-(4.12). Their large-sample properties follow from the delta method and are summarized in the following corollary.

Corollary 4.1. *Suppose that Assumptions 1-5 and Assumptions B.1-B.7 listed in Appendix B hold. Then, as $n \to \infty$,*

(a) for each fixed $y < \gamma_C$, and $v \in \mathcal{P}$,

$$\sqrt{n}\left(\widehat{DMTE}^{avg}(y, v) - DMTE^{avg}(y, v)\right) \xrightarrow{d} N(0, V_{y,v}^{dmte,avg}).$$

(b) for each fixed $\tau \in (0, \overline{\tau}(v, x))$, and $v \in \mathcal{P}$,

$$\sqrt{n}\left(\widehat{QMTE}^{avg}(\tau, v) - QMTE^{avg}(\tau, v)\right) \xrightarrow{d} N(0, V_{\tau,v}^{qmte,avg}).$$

It is also straightforward to construct a weighted-bootstrap confidence interval for these functionals by using $\widehat{w}_x^* = n^{-1}\sum_{i=1}^n \omega_i \, \mathbf{1}\{X_i = x\}$ as weights for the MTE functionals. We omit a detailed description to avoid repetition.

Remark 3. We note that other aggregations of the covariate-specific marginal treatment effect functionals do exist, but may be more challenging to estimate. For instance, one may be interested in functionals of the "unconditional" $DMTR_d(y, v)$, defined as

$$
\begin{aligned}
DMTR_d(y, v) &= \mathbb{E}\left[DMTR_d(y, P, X)|D = d, P = v\right] \\
&= \int DMTR_d(y, v, \bar{x}) f_{X|D,P}(\bar{x}|D = d, P = v) d\bar{x}.
\end{aligned}
$$

It should be clear that $DMTR_d(y, v)$ is different from $DMTR_d^{avg}(y, v)$, though the latter can be more easily estimated as it does not require estimation of conditional densities as the former does. If one were to focus on $DMTR_d(y, v)$, all the uncertainty in estimating it would come from the estimation of the conditional density of X. This follows from Theorem 4.1 establishing that $DMTR_d(y, v, x)$ is $\sqrt{n}$-consistent with discrete X's, while $f_{X|D,P}$ converges at slower rates. Developing a higher-order asymptotic analysis for estimators for $DMTR_d(y, v)$ would be interesting. We leave a detailed analysis of it for future research.

5 Effect of alternative sentences on time-to-recidivism

Our empirical application answers the question: "Do alternative sentences such as fines and community service impact time-to-recidivism in São Paulo, Brazil?". We start our discussion by assessing the plausibility of our identifying assumptions in Subsection 5.1. Subsection 5.2 presents the results of our empirical analysis using our proposed tools. Subsection 5.2 also compares our methods to unpack treatment effect heterogeneity with more traditional methods.

5.1 Assessing the plausibility of our assumptions

The credibility of any causal inference procedure depends on the plausibility of its underlying identification assumptions. In this subsection, we use some descriptive statistics to assess the plausibility of three of our identifying assumptions: Random Assignment (Assumption 1), Monotonicity Condition (Equation (3.1)), and Random Censoring (Assumption 5). We focus on these three assumptions because the others are not controversial in this paper. For instance, Assumptions 2 is plausible as we use the judge's leave-one-out rate of punishment (or "leniency rate") as an instrument for the trial judge's decision, and we have 525 judges and a good amount of variation in this variable as further discussed around Table 4 and Figure 4.

Table 2 focuses on assessing the plausibility of Assumption 1. Although it is not possible to directly test that the instrument (Z) is independent of the potential uncensored time-to-recidivism outcomes $(Y^*(0), Y^*(1))$ and the latent heterogeneity (V) given the censoring variable (C), it is possible to test whether Z is correlated with four excluded covariates: an indicator for whether the defendant has a male name, an indicator for whether the defendant used a public defender instead of hiring a private lawyer, and indicator for whether the defendant was accused of theft, and an indicator for whether the defendant was caught red-handed when committing a crime ("in flagrante delicto").[22] If we find that the instrument is uncorrelated with these four covariates after conditioning on the censoring variable and court-district fixed effects, this could serve as indirect support for Assumption 1.

Table 2 reports the estimated coefficients of regressing the treatment variable (first row) and the instrumental variable (second row) on the four excluded covariates indicated in the columns, the censoring variable and court-district fixed effects. The standard errors are reported in parenthesis and are clusterized at the court district level. We find that our four excluded covariates are correlated with the final ruling in each case (treatment variable), but are not correlated with the punishment rate (instrumental variable). As discussed above, this finding indirectly supports Assumption 1.

Table 3 focuses on assessing the plausibility of the monotonicity condition (Equation (3.1)).

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[22]Appendix C explains how we measure these covariates.

Table 2: Testing the Random Assignment Assumption

	Male Name	Public Defender	Theft	Red-Handed	C
Final Ruling (D)	0.049***	0.161***	0.007	0.132***	0.018***
	(0.009)	(0.023)	(0.015)	(0.011)	(0.004)
Punishment Rate (Z)	0.001	0.003	-0.003	-0.002	-0.001
	(0.002)	(0.008)	(0.005)	(0.003)	(0.001)

Note: The first row reports the estimated coefficients of regressing the treatment variable (D ="punished according to the final ruling in the case") on the binary covariates indicated in the columns, the censoring variable (C is measured in years for readability) and court-district fixed effects. The second row reports the estimated coefficients of regressing the instrumental variable (Z ="trial judge's punishment rate") on the binary covariates indicated in the columns, the censoring variable (C is measured in years for readability) and court-district fixed effects. The standard errors are reported in parenthesis and are clusterized at the court district level. The covariates indicate whether the defendant has a male name, whether the defendant used a public defender instead of hiring a private lawyer, whether the defendant was accused of theft, and whether the defendant was caught red-handed when committing a crime ("in flagrante delicto").

Although it is not possible to directly test that any change in the instrument changes all agents in the same direction with respect to treatment take-up, it is possible to test whether, on average, Z moves agents with different covariate values towards treatment take-up. To implement this test, we adapt the strategy proposed by Bhuller et al. (2020) to our context and regress treatment take-up (final punishment) on the instrument (punishment rate), the censoring variable, and court-district fixed effect for different samples based on values of excluded covariates. We use nine different samples for this test: all defendants, only defendants whose first names are characteristically male, only defendants whose first names are not characteristically male, only defendants who used public defenders, only defendants who hired private lawyers, only defendants accused of theft, only defendants who were not accused of theft, only defendants who were caught red-handed when committing a crime ("in flagrante delicto"), and only defendants who were not caught red-handed. If the estimated punishment rate's coefficient has the same sign in the regressions for all subsamples, this could serve as indirect support for the monotonicity condition (Equation (3.1)).

Table 3 reports the estimated coefficients of regressing the treatment variable (D ="punished according to the final ruling in the case") on the instrumental variable (Z ="trial judge's punishment rate" in Column (1)), the censoring variable (C is measured in years for readability in Column (2)) and court-district fixed effects for the sample indicated in the rows. The standard errors are reported in parenthesis and are clusterized at the court district level. Column(3) reports the number of observations in each sample. We find that the estimated punishment rate's coefficient has the same sign and similar magnitude in the regressions for all subsamples, indirectly supporting the monotonicity condition (Equation (3.1)).

Figure 3 focuses on assessing the plausibility of Assumption 5. Although it is not possible to directly test that the potential uncensored time-to-recidivism outcomes $Y^*(0)$ and $Y^*(1)$ are independent of the censoring variable C (time-to-follow-up) given the value of the latent heterogeneity (or punishment resistance) V, it is possible to test whether the realized uncensored

Table 3: Testing the Monotonicity Condition

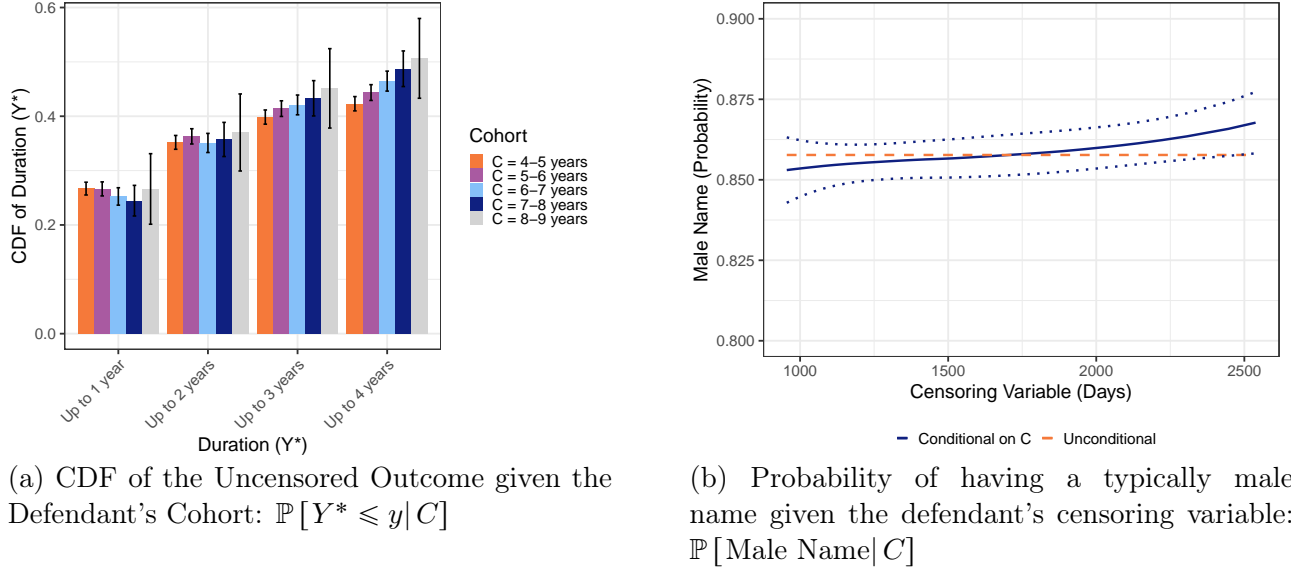|	Punishment Rate (Z)	Censoring Variable (C)	Sample Size
	(1)	(2)	(3)
(1) All	0.772***	0.013***	43,468
	(0.046)	(0.004)	
(2) Male	0.761***	0.012***	37,304
	(0.051)	(0.004)	
(3) Female	0.799	0.020***	6,164
	(0.080)	(0.006)	
(4) Public Defender	0.718***	0.021**	5,101
	(0.152)	(0.010)	
(5) Private Lawyer	0.766***	0.022***	38,367
	(0.054)	(0.004)	
(6) Theft	0.765***	0.031***	24,200
	(0.071)	(0.005)	
(7) Other Crimes	0.750***	-0.006	19,268
	(0.082)	(0.005)	
(8) Red-Handed	0.766***	0.030***	15,147
	(0.076)	(0.005)	
(9) Not Red-Handed	0.789***	0.005	28,321
	(0.065)	(0.004)	

Note: This table reports the estimated coefficients of regressing the treatment variable (D ="punished according to the final ruling in the case") on the the instrumental variable (Z ="trial judge's punishment rate" in Column (1)), the censoring variable (C is measured in years for readability in Column (2)) and court-district fixed effects for different samples. The standard errors are reported in parenthesis and are clusterized at the court district level. The first row uses our entire sample, the second row uses only defendants whose first names are characteristically male, the third row uses only defendants whose first names are not characteristically male, the fourth row uses only defendants who used public defenders, the fifth row uses only defendants who hired private lawyers, the sixth row uses only defendants accused of theft, the seventh row uses only defendants who were not accused of theft, the eight row uses only defendants who were caught red-handed when committing a crime ("in flagrante delicto"), and the nineth row uses only defendants who were not caught red-handed. Column(3) reports the number of observations in each sample.

time-to-recidivism outcome (Y^*) is (approximately) independent of C. If we find no strong evidence against the independence assumption between Y^* and C, this could serve as indirect support for (or lack of support against) Assumption 5.

Figure 3a shows the cumulative distribution function (CDF) of the uncensored time-to-recidivism outcome given cohorts defined based on the time-to-follow-up variable. If Y^* is independent of C, then this CDF should not vary across cohorts. Taking into account the sampling uncertainty, Figure 3a shows that the CDFs are not very different across censoring cohorts, indirectly suggesting that the censoring variable may be independent of the potential outcomes as required by Assumption 5.[23]

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[23]This figure also suggests that if there is dependence, the potential outcomes would be negatively regression dependent on the censoring variable, as discussed in Appendix G.1. Moreover, as a robustness check against violations of the random censoring Assumption, we re-do our analysis focusing solely on the period starting in 2014. These results are presented in Appendix D.2 and are similar to the ones using the entire sample period, indicating that our results are robust to possible violations of the Random Censoring Assumption.

Figure 3: Assessing the Plausibility of Assumption 5

(a) CDF of the Uncensored Outcome given the Defendant's Cohort: $\mathbb{P}\left[Y^{*} \leqslant y \mid C\right]$

(b) Probability of having a typically male name given the defendant's censoring variable: $\mathbb{P}\left[\text{Male Name} \mid C\right]$

Notes: Figure 3a shows the cumulative distribution function of the uncensored potential outcome (Y^{*}) given cohorts based on the censoring variable. Each color denotes a different cohort: orange denotes defendants who are observed for at least four years and at most five years during our sampling period, purple denotes defendants who are observed for at least five years and at most six years, light blue denotes defendants who are observed for at least six years and at most seven years, dark blue denotes defendants who are observed for at least seven years and at most eight years, and gray denotes defendants who are observed for at least eight years and at most nine years. These conditional CDFs are evaluated at four values of the uncensored potential outcome (one, two, three, or four years), and these evaluation points are denoted in the x-axis. The y-axis denotes the value of the CDF, while black lines denote point-wise 99%-confidence intervals around the values of the CDF.

Figure 3b shows, as a solid dark blue line, the probability that the defendant has a typically male name as a function of their censoring variable. This nonparametric function was estimated using a local linear regression with an Epanechnikov kernel based on Calonico, Cattaneo and Farrell (2019). The bandwidth was optimally selected according to the IMSE criterion. The dotted dark blue lines are robust bias-corrected 99%-confidence intervals. The dashed orange line is the unconditional probability of having a typically male name.

There are other ways to assess the plausibility of the random censoring assumption, too. Another practical test we can entertain is to analyze the relationship between the censoring variable and an excluded covariate: an indicator for whether the defendant has a male name. Under random censoring, we would expect to see that the information about C should not affect the distribution of the excluded covariate. Figure 3b entertains this exercise and shows the probability of having a typically male name given the defendant's censoring variable (dark blue line). Regardless of the censoring variable, this probability is relatively close to the unconditional share of male names (orange line). This, again, can serve as indirect evidence of the plausibility of our random censoring assumption.

5.2 Empirical results

In this section, we present our empirical results. Section 5.2.1 contains the information about the first stage of our estimation procedure, Equation (4.1), which relates how censoring, court district dummies, and the judge's leniency rate affects the final ruling of the case. Section 5.2.2 presents our estimates of the causal parameters in Corollary 4.1.

5.2.1 First stage results

We start by presenting the results of the first stage regression in our empirical analysis. In our model, the treatment variable D ("final ruling") is a function of the instrument Z ("trial judge's punishment rate"), the censoring variable C, and court district fixed effects. Following Subsection 4.2, we use a polynomial series to approximate the propensity score and report the estimated coefficients of a quadratic model in Table 4. Note that our instrument is strong according to the F-statistic of the first-stage regression. This result suggests that Assumption 2 is plausible.

Table 4: First Stage Results

	Z	Z^2	C
Coefficient	0.663***	0.096	0.012***
Clusterized S.E.	(0.235)	(0.208)	(0.004)
F-statistic		817	

Note: The left-hand side variable is our treatment variable, i.e., D ="punished according to the final ruling in the case". The standard errors are clusterized at the court district level. The third line reports the F-Statistic of a hypothesis test whose null is that the coefficients associated with Z and Z^2 are equal to zero. The first stage regression controls for court district fixed effects. To improve readability, we multiply the coefficient of the censoring variable (and its standard error) by 365. This transformation is equivalent to measuring the censoring variable in years instead of days.

We also report the distribution of the estimated propensity score in Figure 4. The blue histogram shows the distribution of the estimated propensity score given that defendant was punished (treated group) while the white histogram shows the distribution of the estimated propensity score given that defendant was not punished (comparison group). We find that most defendants have a probability of being punished around 50%. However, some defendants are more unlikely to be punished (estimated propensity score around 30%) and others are more likely to be punished (estimated propensity score around 70%). These widely spread propensity score distributions are positive for identification and estimation because they allow us to discuss DMTE and QMTE functions evaluated at many different points of the latent heterogeneity variable.
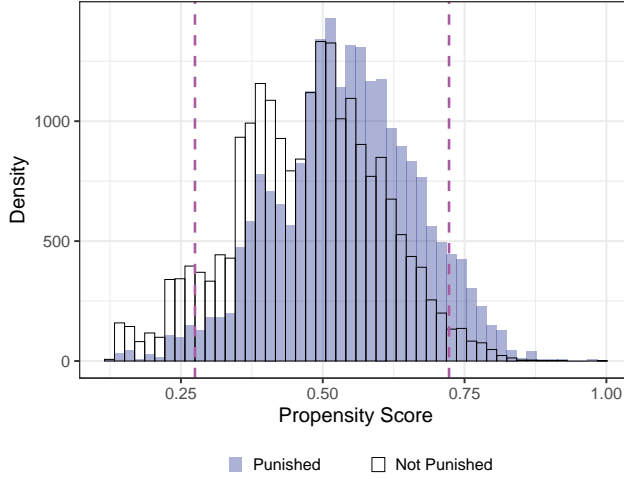
The vertical lines in Figure 4 denote the unconditional 5th and 95th percentiles of the estimated propensity score. When discussing our results about the DMTE and QMTE functions, we only report the estimates for latent heterogeneity values between these two percentiles. We do so to avoid extrapolation bias and to ensure the plausibility of Assumption 4.

5.2.2 Estimated causal parameters

To estimate the DMTE and QMTE functions in our empirical application, we flexibly account for court district fixed effects. More precisely, we estimate 193 district-specific functions for each of our treatment effect parameters (Theorem 4.1). Although very flexible, this strategy makes it challenging to concisely report summary results. The way we proceeded was to

Figure 4: Distribution of the estimated propensity score among treatment groups

Notes: The blue histogram shows the distribution of the estimated propensity score among defendants who were punished (treated group). The white histogram shows the distribution of the estimated propensity score among defendants who were not punished (comparison group). The vertical lines denote the unconditional 5th and 95th percentiles of the estimated propensity score.

average these district-specific functions over court districts using the proportion of cases per court district as weights, as in Corollary 4.1. We report the average DMTE function in Section 5.2.2.1 and the average QMTE function in Section 5.2.2.2. Moreover, we compare our proposed methods against standard methods in the literature in Part 5.2.2.3.

5.2.2.1 Estimated DMTE function

Figure 5 shows the estimated average $DMTE(y, \cdot)$ functions for $y \in \{1, 2, \ldots, 8\}$, where instead of measuring time-to-recidivism in days we measured it in years (to enhance readability).[24] These point estimates show relevant heterogeneity with respect to the treatment resistance (horizontal axis denotes values of V) and with respect to the recidivism horizon (different colors denote different values of y).

First, the $DMTE(y, \cdot)$ functions are increasing. This functional behavior indicates that defendants whom almost all judges would punish are less likely to recidivate, while defendants who would be punished only by tough judges are more likely to recidivate compared to situations in which they would not be punished.[25] This conclusion is supported by our 90%-confidence intervals (Figures D.2a-D.3d). Importantly, these point-wise confidence intervals suggest that constant treatment effects are implausible in our empirical context. Hence, they highlight the importance of taking idiosyncratic latent heterogeneity seriously through an analysis of "MTE-

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[24]In our data, we observe time-to-recidivism in days. To illustrate the readability improvements of writing the $DMTE$ function in years instead of days, we focus on one value of the time-to-recidivism variable. The $DMTE(y, \cdot)$ function when $y = 2$ shows the distributional marginal treatment effect given by $\mathbb{P}[Y^*(1) \leqslant 2 \cdot 365 \text{ days} \mid V = v] - \mathbb{P}[Y^*(0) \leqslant 2 \cdot 365 \text{ days} \mid V = v]$.

[25]Appendix G.2 shows that these results are robust to violations of Assumption 5.

Figure 5: $DMTE\left(y,\cdot\right)$ for $y \in \{1, 2, \ldots, 8\}$
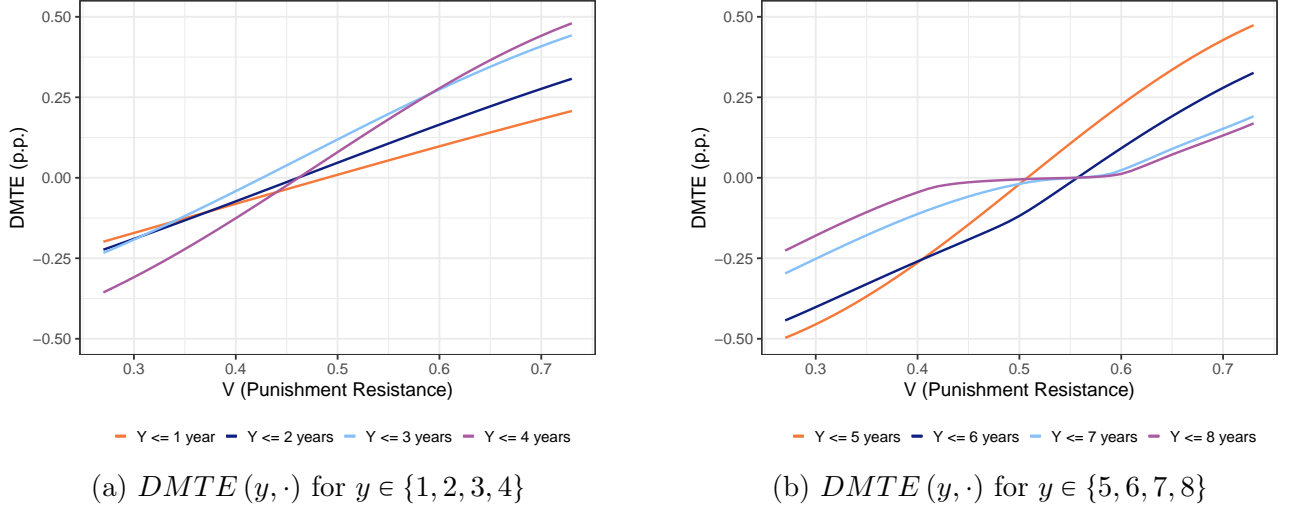
(a) $DMTE\left(y,\cdot\right)$ for $y \in \{1, 2, 3, 4\}$ (b) $DMTE\left(y,\cdot\right)$ for $y \in \{5, 6, 7, 8\}$

Notes: Solid lines are the point estimates for the average $DMTE\left(y,\cdot\right)$ functions indicated in the legend of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported in Appendix D.1. These confidence intervals were computed using the weighted bootstrap, clustered at the court district level (Subsection 4.2).

like" parameters.

Second, the $DMTE\left(y,\cdot\right)$ functions are steeper for $y \in \{3, 4, 5, 6\}$ than for $y \in \{1, 2, 7, 8\}$. This functional behavior indicates that the effect of alternative sentences on recidivism is more intense for extreme cases (small or large punishment resistance levels) in the mid-run than it is in the short or long-run horizon.

This rich heterogeneity illustrates the importance of considering different treatment resistance levels. Our point estimates suggest that designing sentencing guidelines that encourage strict judges to become more lenient could increase time to recidivism. However, the $DMTE$ functions do not allow us to quantify this impact directly.

For this reason, $DMTE$ functions may not be the ideal way to convey the main takeaway of the application, even though they answer well-posed and policy-relevant questions. In what follows, we show that this limitation can be minimized by focusing on other functionals of interest, such as the QMTE, which are measured in days instead of percentage points.
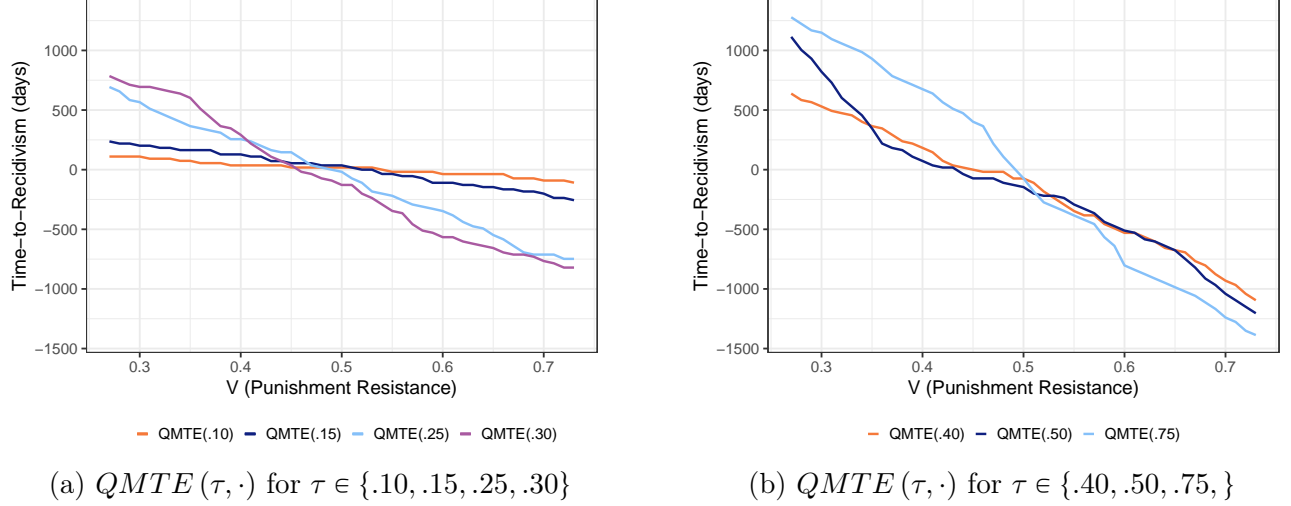
5.2.2.2   Estimated QMTE function

To better understand the time trade-offs associated with the effect of punishment on time to recidivism, we now focus on the average quantile marginal treatment effect functions. These functionals are easier to interpret than the DMTE functions because they express the underlying treatment effects in the same units as the time-to-recidivism outcomes, i.e., days before the first recidivism event.

Figure 6 shows the estimated average QMTE($\tau, \cdot$) functions for $\tau \in \{.10, .15, .25, .30, .40, .50, .75\}$.

Once more, these point estimates show relevant heterogeneity with respect to the punishment resistance (horizontal axis denotes values of V).

Figure 6: $QMTE(\tau, \cdot)$ for $\tau \in \{.10, .15, .25, .30, .40, .50, .75\}$

(a) $QMTE(\tau, \cdot)$ for $\tau \in \{.10, .15, .25, .30\}$ (b) $QMTE(\tau, \cdot)$ for $\tau \in \{.40, .50, .75, \}$

Notes: Solid lines are the point estimates for the average $QMTE(\tau, \cdot)$ functions indicated in the legend of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported in Appendix D.1. These confidence intervals were computed using the weighted bootstrap clusterized at the court district level (Subsection 4.2).

Although the level of the estimated $QMTE(\tau, \cdot)$ functions depends on the quantile, all functions are decreasing in the unobserved resistance to punishment. These point estimates suggest that defendants whom almost all judges would punish would take longer to recidivate when punished. In contrast, defendants who would be punished only by tough judges would recidivate faster compared to situations in which they would not be punished. This result is statistically significant for $\tau \in \{.10, .15, .25, .30, .40, .50, .75\}$ at the 10% significance level, according to Figures D.4 and D.5a-D.5c in the Appendix. Interestingly, these point-wise confidence intervals suggest that constant treatment effects are likely invalid in our empirical setting. Hence, they emphasize the importance of taking essential heterogeneity seriously through an analysis of "MTE-like" parameters.

We reach a similar conclusion when we analyze the $QMTE(\cdot, v)$ as a function of the quantiles for specific values of unobserved resistance to treatment. Figure D.1 in the Appendix shows the average $QMTE(\cdot, v)$ for $v \in \{.3, .4, .5, .6, .7\}$. Our results suggest that this function is always positive for small values of the unobserved resistance to punishment, while it is always negative for large values of v.

Overall, our QMTE point estimates suggest that designing sentencing guidelines that encourage strict judges to become more lenient could increase time-to-recidivism.

### 5.2.2.3 Comparison with other available methods

Here, we compare our proposed methods against other available methods in the literature. Differently from our approach, these estimates ignore that the outcome variable is right-censored and provide different conclusions when compared against our proposed estimator. For brevity, we focus our attention on the effects on the 25th and 50th percentiles ($QMTE\left(.25, \cdot\right)$ and $QMTE\left(.50, \cdot\right)$ functions) in Figure 7.

Figure 7 focuses on the effect on the 25th and 50th percentiles of our outcome variable. Our proposed methods are illustrated by the purple lines. We have the average $QMTE\left(.25, \cdot\right)$ function in Figure 7a and the average $QMTE\left(.50, \cdot\right)$ function in Figure 7b (Corollary 4.1). The light blue lines denote a "naive" version of our estimators that follows the same steps as described in Section 4.2, but do not condition on the censoring variable. The orange lines denote the standard method in the IV literature that accounts for endogenous selection into treatment but ignores (or aggregates) treatment effect heterogeneity with respect to unobserved resistance to treatment. The orange line in Figure 7a is the treatment coefficient of an IV quantile regression (Kaplan and Sun, 2017) for the 25th percentile, while Figure 7b is the treatment coefficient of an IV quantile regression (Kaplan and Sun, 2017) for the 50th percentile. Both IV quantile regressions use the censored outcome variable as the left-hand side variable, control for court district fixed effects, and use the judge's punishment rate as the instrument for the defendant being punished.

Figure 7: Comparing our Proposed Methods against Other Available Methods
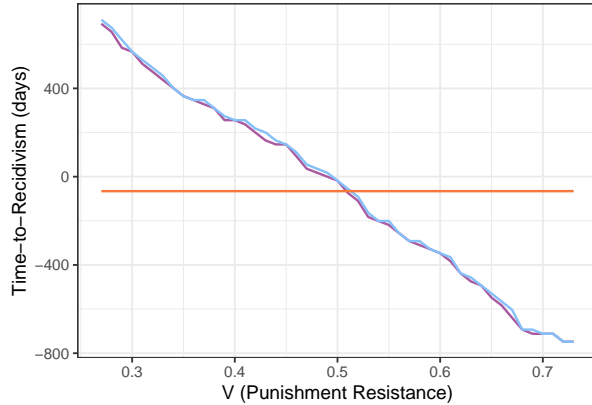
(a) $QMTE\left(.25, \cdot\right)$ against Other Available Methods
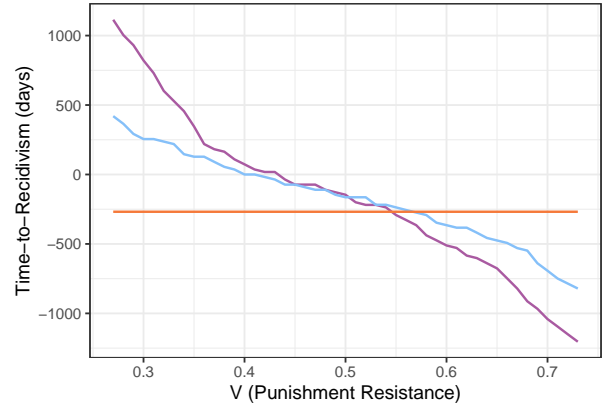
(b) $QMTE\left(.50, \cdot\right)$ against Other Available Methods

Notes: Our proposed methods are illustrated by the purple lines. We have the average $QMTE\left(.25, \cdot\right)$ function in Figure 7a and the average average $QMTE\left(.50, \cdot\right)$ function in Figure 7b (Corollary 4.1). The light blue lines denotes a naive version of our estimators that ignores censoring. The orange lines are the treatment coefficients of IV quantile regressions (Kaplan and Sun, 2017) for the 25th and 50th percentiles.

Analyzing Figures 7a and 7b, we find that the IV quantile regression does not capture the

rich heterogeneity behind the treatment effects of fines and community service. In particular, the IV quantile regression estimates suggest a negative effect, ignoring that the treatment increases time-to-recidivism for some defendant types. Importantly, the IV quantile regression estimates do not lie entirely within the 90%-confidence intervals of the correctly estimated $QMTE\,(.25,\cdot)$ and $QMTE\,(.50,\cdot)$ functions (Figures D.4c and D.5b).

Moreover, in Figure 7a, we observe that our proposed estimator (purple line) and its naive version (light blue line) reach similar point estimates. This finding is unsurprising because the estimated $QMTR_d\,(.25,\cdot)$ functions are always smaller than 2.5 years, and all defendants are observed for at least 2 years. Consequently, the censoring problem is not binding for low percentiles. However, the censoring problem is binding for higher percentiles. In Figure 7b, we focus on the $QMTR_d\,(.50,\cdot)$ function and find that our proposed estimator (purple line) and its naive version (light blue line) differ in relevant ways. For example, the naive estimator finds a $QMTE$ function that is less steep, implying smaller treatment effects for extreme values of punishment resistance. Importantly, the naive estimates do not lie entirely within the 90%-confidence intervals of the correctly estimated $QMTE\,(.50,\cdot)$ function (Figure D.5b).

All in all, our results indicate that our proposed tools can provide detailed measures of treatment effect heterogeneity of punishing misdemeanor offenses on time-to-recidivism that other methods are not meant to capture.

6 Conclusion

In this paper, we identify the distributional marginal treatment effect ($DMTE$) and the quantile marginal treatment effect ($QMTE$) functions when the outcome variable is right-censored. To do so, we extend the MTE framework (Heckman et al., 2006; Carneiro and Lee, 2009) to scenarios with duration outcomes. In this section, we discuss in which contexts our proposed methodology can be used and deepen our empirical discussion.

Our methodology can be applied to many empirical problems that face two simultaneous identification challenges: endogenous selection into treatment and right-censored data. In our empirical application, we focus on the effect of misdemeanor punishment on defendants' time-to-recidivism. In this case, judges observe more information than the econometrician when making their decisions, and time-to-recidivism is a right-censored variable. A similar problem arises when analyzing the effect of rehabilitation programs on recidivism (Alsan, Barnett, Hull and Yang, 2024). Furthermore, the same identification challenges appear in labor economics when analyzing the effect of receiving unemployment benefits on unemployment spells (Chetty, 2008; Delgado et al., 2022) or in development economics when analyzing the impact of micro-credit on firm survival (Banerjee, Karlan and Zinman, 2015). Moreover, in the health sciences, when studying the effect of a drug on survival time, a researcher has to address both identifi-

cation problems too (Sullivan, Zwaag, El-Zeky, Ramanathan and Mirvis, 1993; Spiegel, 2002; Trinquart, Jacot, Conner and Porcher, 2016).

Concerning its empirical contribution, our work is inserted in the literature about the effect of fines and community service sentences on future criminal behavior. Five recent papers in this field were written by Huttunen et al. (2020), Giles (2023), Klaassen (2021), Possebom (2023), Lieberman et al. (2023). They all focus on binary variables indicating recidivism within a pre-specified period. Huttunen et al. (2020) and Giles (2023) find that this type of punishment increases the probability of recidivism in Finland and Milwaukee (a city in the State of Wisconsin in the U.S.), respectively. Klaassen (2021) finds that alternative sentences decrease the probability of recidivism in North Carolina (a state in the U.S.). Possebom (2023) finds that this type of punishment has a small and statistically insignificant effect on the probability of recidivism in São Paulo, Brazil. Finally, Lieberman et al. (2023) analyzes five American states and finds that court fees do not impact recidivism.

Differently from these five papers, our outcome variable is time-to-recidivism. Using a continuous outcome instead of binary indicators allows for a finer analysis of the heterogeneous effects of fines and community service sentences on future criminal behavior, and may conciliate the conflicting results in the previous literature. For example, we find that this type of punishment increases time-to-recidivism for some individuals while decreasing it for other individuals. If the first type of individual is more common in North Carolina than in Milwaukee and Finland, our focus on essential heterogeneity may shed light on these conflicting results.

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Was Javert right to be suspicious?
Unpacking treatment effect heterogeneity of alternative sentences on time-to-recidivism in Brazil

Online Appendix

Santiago Acerenza   Vitor Possebom   Pedro H. C. Sant'Anna

January 8, 2025

This online appendix contains proofs and additional results for the paper "Was Javert right to be suspicious? Unpacking treatment effect heterogeneity of alternative sentences on time-to-recidivism in Brazil" by Santiago Acerenza, Vitor Possebom, and Pedro H. C. Sant'Anna h. Section A contains proofs and auxilliary lemmas for results stated in the main text. Section B contains additional details on standard regularity conditions for our proposed identification and estimation results to work. Section C explains the construction of our dataset in detail. Section D presents additional results for our empirical application. Section E provides additional motivation and theoretical arguments for focusing on duration (time-to-event) outcomes instead of short-term outcomes. Section F discusses how one can allow for additional dependence between the censoring variable and the potential outcomes and still achieve point-identification. Finally, Section G extends our results to some types of dependent censoring and displays partial identification of the causal parameters of interest.

A   Proofs of the main results

We start by stating an auxiliary lemma that will be used to derive our main identification results.

Lemma A.1. *If Assumptions 1-4 hold, then, for any $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$,*

$$\mathbb{P}\left[Y \leqslant y, D = 1 \mid P\left(Z, C\right) = v, C = y + \delta\right] = \int_0^v \mathbb{P}\left[Y^*(1) \leqslant y \mid C = y + \delta, V = v\right] dv \quad \text{(A.1)}$$

and

$$\mathbb{P}\left[Y \leqslant y, D = 0 \mid P\left(Z, C\right) = v, C = y + \delta\right] = \int_v^1 \mathbb{P}\left[Y^*(0) \leqslant y \mid C = y + \delta, V = v\right] dv, \quad \text{(A.2)}$$

If Assumption 5 holds too, then, for any $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$,

$$\mathbb{P}\left[Y \leqslant y, D = 1 \mid P\left(Z, C\right) = v, C = y + \delta\right] = \int_0^v \mathbb{P}\left[Y^*(1) \leqslant y \mid V = v\right] dv \quad \text{(A.3)}$$

and

$$\mathbb{P}\left[Y \leqslant y, D = 0 \mid P\left(Z, C\right) = v, C = y + \delta\right] = \int_v^1 \mathbb{P}\left[Y^*(0) \leqslant y \mid V = v\right] dv. \quad \text{(A.4)}$$

A.1   Proof of Lemma A.1

Fix $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$. To prove (A.1), note that

$$\mathbb{P}\left[Y \leqslant y, D = 1 \mid P\left(Z, C\right) = v, C = y + \delta\right]$$

$$= \mathbb{E}\left[\mathbf{1}\left\{Y \leqslant y\right\} \mathbf{1}\left\{P\left(Z, C\right) \geqslant V\right\} \mid P\left(Z, C\right) = v, C = y + \delta\right]$$

by (3.1)

$$= \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\} \mathbf{1}\left\{v \geqslant V\right\} \mid P\left(Z, y + \delta\right) = v, C = y + \delta\right]$$

because $\mathbf{1}\left\{Y(1) \leqslant y\right\} = \mathbf{1}\left\{Y^*(1) \leqslant y\right\}$ when $C > y$

$$= \int_0^1 \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\} \mathbf{1}\left\{v \geqslant \tilde{v}\right\} \mid P\left(Z, y + \delta\right) = \tilde{v}, C = y + \delta, V = \tilde{v}\right] d\tilde{v}$$

by the Law of Iterated Expectations and Assumption 3

$$= \int_0^1 \mathbf{1}\left\{v \geqslant \tilde{v}\right\} \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\} \mid P\left(Z, y + \delta\right) = v, C = y + \delta, V = \tilde{v}\right] d\tilde{v}$$

$$= \int_0^v \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\} \mid P\left(Z, y + \delta\right) = v, C = y + \delta, V = \tilde{v}\right] d\tilde{v}$$

$$= \int_0^v \mathbb{P}\left[Y^*(1) \leqslant y \mid C = y + \delta, V = \tilde{v}\right] d\tilde{v}$$

by Assumption 1.

We can prove (A.2) analogously.

To prove (A.3), observe that

$$\mathbb{P}\left[Y \leqslant y, D = 1 \middle| P(Z,C) = v, C = y + \delta\right]$$

$$= \int_0^v \mathbb{P}\left[Y^*(1) \leqslant y \middle| C = y + \delta, V = \tilde{v}\right] d\tilde{v}$$

$$= \int_0^v \mathbb{P}\left[Y^*(1) \leqslant y \middle| V = \tilde{v}\right] d\tilde{v}$$

by Assumption 5.

We can prove (A.4) analogously.

A.2 Proof of Proposition 3.1

Fix $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

First, note that Equations (A.3) and (A.4) imply that

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 1 \middle| P(Z,C) = v, C = y + \delta\right]}{\partial v} = \mathbb{P}\left[Y^*(1) \leqslant y \middle| V = v\right] \tag{A.5}$$

and

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 0 \middle| P(Z,C) = v, C = y + \delta\right]}{\partial z} = -\mathbb{P}\left[Y^*(0) \leqslant y \middle| V = v\right] \tag{A.6}$$

according to the Leibniz Integral Rule.

Combining Equations (2.1) and (A.5)-(A.6), we prove that

$$DMTR_d(y, v) = (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P(Z,C) = v, C = y + \delta\right]}{\partial v}$$

for any $d \in \{0, 1\}$.

Since the last equation holds for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$DMTR_d(y, v) = (2d - 1) \cdot \int_y^{+\infty} \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P(Z,C) = v, C = c\right]}{\partial v} \cdot f_{C|P(Z,C)=v,C>y}(c) \, dc$$

for any $d \in \{0, 1\}$.

B Regularity Conditions and Semiparametric Estimation

In this appendix we elaborate on standard regularity conditions for our proposed identification and estimation results to work.

B.1 Main Regularity Conditions

Although identification does not rely on any parametric assumption, some of them aid the estimation procedure. Covariates are easily incorporated when semiparametric assumptions are made and the curse of dimensionality is avoided. Additionally, semiparametric assumptions demand less data. In this appendix, we follow Rothe (2009) closely, but adapt his setting for the case where the link function is known instead of unknown. For the rest of the section, we assume an *i.i.d* sample. In this context, we introduce the following assumption:

Assumption B.1 (Semiparametric CDF). *Let* $\mathbb{P}\left[Y \leqslant y, D = d \mid P, C, X\right] = \Lambda(\beta_0\left(d, y\right) + \beta_C\left(d, y\right) C + \beta_P\left(d, y\right) P + X'\beta_X\left(d, y\right))$, *where* $\Lambda()$ *is a known link function up to a finite dimensional vector (such as the logistic link), which is continuously differentiable in the index. Let* $\Lambda'(.)$ *be the derivative of* $\Lambda(.)$, *which is continuous.*

For the sake of exposition, let $W_{y,d} = 1\{Y \leqslant y, D = d\}$, $H = \{1, C, P, X\}$, $\hat{H} = \{1, C, \hat{P}, X\}$, $H_v = \{1, C, v, X\}$, $\beta_{d,y} := (\beta_0\left(d, y\right), \beta_C\left(d, y\right), \beta_P\left(d, y\right), \beta_X\left(d, y\right))$ for any y and $d \in \{0, 1\}$. Taking the derivative with respect to P for $\Lambda(\cdot)$ for both $W_{y,1}$ and $W_{y,0}$, we get the $DMTE(y, v)$ as

$$DMTR_1(y, v) - DMTR_0(y, v) = \Lambda'(\beta_{1,y}H_v)\beta_P\left(1, y\right) - \Lambda'(\beta_{0,y}H_v)\beta_P\left(0, y\right)$$

If P was known, it would be easy to estimate the DMTE as in the parametric part.

Since P is not known, we can estimate P in a semiparametric first stage, and obtain estimates for $\beta_{d,y}$ from the following maximum-likelihood procedure.[26] We focus on $d = 1$ for the sake of exposition and denote the semiparametric first-stage estimates by $\hat{P}$. Define

$$Ln(\beta_{1,y}, \hat{P}) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i W_{y,1,i} log[\Lambda(\beta_{1,y}\hat{H}_i)] + (1 - W_{y,1,i}) log[1 - \Lambda(\beta_{1,y}\hat{H}_i)] \tag{B.1}$$

with solution $\hat{\beta}_{1,y}(\hat{P})$. If P was known, we could use the following unfeasible standard maximum likelihood procedure:

$$Ln(\beta_{1,y}, P) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i W_{y,1,i} log[\Lambda(\beta_{1,y}H_i)] + (1 - W_{y,1,i}) log[1 - \Lambda(\beta_{1,y}H_i)] \tag{B.2}$$

with solution $\hat{\beta}_{1,y}(P)$.

To analyze our semiparametric estimator (B.1), we need to ensure that the unfeasible estimator in (B.2) is well-behaved. To do so, we impose the following assumption:

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[26]In the semiparametric first stage, we can estimate P using a standard series estimator.

A-4

Assumption B.2 (Unfeasible Likelihood). *The maximum likelihood estimator of* (B.2), *follows standard regularity conditions from Newey and McFadden (1994) for consistency and asymptotic normality.*

Assumption B.2 ensures that standard parametric inference could be performed if P was observed, implying that $\hat{\beta}_{1,y}(P) \xrightarrow{p} \beta_{1,y}$. Since Λ is the logistic link, the result is standard.

To ensure that our semiparametric estimator is consistent and derive its asymptotic distribution, we need to ensure that our propensity score estimator converges sufficiently fast and satisfies some regularity conditions. To do so, we follow Rothe (2009) and impose the following assumption.

Assumption B.3 (First stage assumptions). *Let $\hat{P}$ satisfy:*

1. *$\hat{P}_i - P_i = \frac{1}{N} \sum_j w_n(Z_i, C_i, X_i, Z_j, C_j, X_j) \phi_j + r_{in}$ with $\max_i \|r_{in}\| = o_p(N^{-\frac{1}{2}})$ and $\max_i |\hat{P}_i - P_i| = o_p(N^{-\frac{1}{4}})$ where $\phi_j = \phi(D_j, Z_j, C_j, X_j)$ is an influence function with $\mathbb{E}[\phi_j | Z_j, C_j, X_j] = 0$ and $\mathbb{E}[\phi_j^2 | Z_j, C_j, X_j] < \infty$ and weights $w_n(Z_i, C_i, X_i, Z_j, C_j, X_j) = o(N)$.*

2. *There exists a space $\mathcal{P}$ such that $\mathbb{P}(\hat{P} \in \mathcal{P}) \to 1$ and $\int_0^\infty \sqrt{\log N(\lambda, \mathcal{P}, \|\cdot\|_\infty)} d\lambda < \infty$ where $N(\lambda, \mathcal{P}, \|\cdot\|_\infty)$ is the covering number with respect to the L_∞ norm of the class of functions $\mathcal{P}$.*

Assumption B.3 is analogous to Assumption 8 in Rothe (2009) and can be interpreted as a high-level condition on the propensity score estimator. Assumption B.3(i) states that the estimator admits a certain asymptotic expansion. Assumption B.3(ii) requires the propensity score estimator to take values in some well-behaved function space with probability approaching 1. These are relatively mild conditions. Indeed, it is easy to show that series-based and kernel-based estimators satisfy Assumption B.3; see, e.g., Rothe (2009, page 55), for a discussion.

B.2 Additional Regularity Conditions

Besides the previously mentioned conditions, which are the key components of the semiparametric procedure, we need to add additional regularity conditions to ensure that our procedures work.

Assumption B.4. *Assume that $\mathbb{P}[Y \leq y, D = d | P, C, X], \mathbb{P}[D = d | P, C, X]$ are twice continuously differentiable in P.*

Assumption B.5. *Assume that the support of Z is known and is a Cartesian product of compact connected intervals on which Z has a probability density function that is bounded away from zero.*

Assumption B.6. *Assume that $\psi_l(z)$, for $l \in L$ are r_ψ-times continuously differentiable on the support of Z for $r_\psi \geqslant 2$, where $\psi_l(z)$ is used to approximate $\varphi(z)$, an unknown function.*

Assumption B.7. *Assume that $Y^*(d)$ is continuous with respect to the Lebesgue measure.*

Assumption B.4 assures that we can apply the Leibniz Integral Rule to identify the $DMTR$ functions. Assumptions B.5 and B.6 are standard in the series estimation literature. In particular, Assumption B.6 implies that the asymptotic bias (due to the series approximation by regression splines) converges to zero at a rate of L^{-r_ψ} as the number of approximation functions, L, diverges to infinity. Assumption B.7 is a regularity condition that ensures point identification for our quantile results.

B.3 Proof of Theorems 4.1 and H.1: Consistency and Asymptotic Normality

To ensure our estimators of the $DMTE, QMTE, MTE$ are consistent and asymptotically normal, we first need to ensure consistency of the feasible estimator of all the components of the DMTE. Then, we use functional approximation results to show asymptotic results for $DMTE, QMTE, MTE$.

B.3.1 Consistency of the $\beta_{d,y}$ estimators

We need to prove asymptotic equivalence between the solution of Equations (B.1) and (B.2). Then, by Assumption B.2, we get the consistency of the feasible semiparametric estimator. Note that

$$
\begin{aligned}
\sup_{\beta_{1,y}} & |Ln(\beta_{1,y}, \hat{P}) - Ln(\beta_{1,y}, P)| \\
& \leqslant \left[\inf_{\beta_{1,y}} \min_i \{\Lambda(\beta_{1,y}\hat{H}_i), \Lambda(\beta_{1,y}H_i), 1 - \Lambda(\beta_{1,y}\hat{H}_i), 1 - \Lambda(\beta_{1,y}H_i)\} \left(\sup_{\beta_{1,y}} \max_i |\Lambda(\beta_{1,y}\hat{H}_i) - \Lambda(\beta_{1,y}H_i)| \right) \right] \\
& \leqslant \left[O(1) \left(\sup_{\beta_{1,y}} \max_i |\Lambda(\beta_{1,y}\hat{H}_i) - \Lambda(\beta_{1,y}H_i)| \right) \right] \\
& = o_p(1),
\end{aligned}
$$

where the first inequality can be derived using standard algebraic manipulations. Moreover, the second inequality holds because $\Lambda(\cdot) \in (0,1)$. Furthermore, note that $\Lambda(\cdot)$ is continuous and $\max_i |\hat{H}_i - H_i|$ converges due to Assumption B.3, implying that $\max_i |\Lambda(\beta_{1,y}\hat{H}_i) - \Lambda(\beta_{1,y}H_i)|$ converges due to the continuous mapping theorem. Finally, since the supremum over $\beta_{1,y}$ in the third line is also continuous, we can apply the continuous mapping theorem again to prove the last equality.

Furthermore, $Ln(\beta_{1,y}, P)$ is a standard parametric likelihood, implying that it converges uniformly in $\beta_{1,y}$ to its expectation (Newey and McFadden, 1994, Lemma 2.4). Formally, we

have that

$$\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, P) - L(\beta_{1,y})| = o_p(1)$$

where $L(\beta_{1,y}) = \mathbb{E}\left[Ln(\beta_{1,y}\right] = \mathbb{E}\left[W_{y,1,i}log(\Lambda(\beta_{1,y}H)) + (1 - W_{y,1,i})log(1 - \Lambda(\beta_{1,y}H))\right]$ is a non-random function that is continuous in $\beta_{1,y}$. Taken together, it follows from the triangle inequality that

$$\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, \hat{P}) - L(\beta_{1,y})| = o_p(1)$$

implying that $\hat{\beta}_{1,y}(P)$ is consistent whenever $L(\beta_{1,y})$ attains a unique maximum at the true value of the parameter, which is the case by our identification results and Assumption B.1.

As a consequence, the consistency of our feasible semiparametric estimator follows from Theorem 2.1 by Newey and McFadden (1994) via Assumption B.2.

B.3.2   Asymptotic distribution of the $\beta_{d,y}$ estimators

Now, we derive the asymptotic distribution of our semiparametric estimator in (B.1). Let $Ln(\beta_{1,y}, \hat{P}_i)_\beta, Ln(\beta_{1,y}, P_i)_\beta, L(\beta_{1,y}, P_i)_\beta$ be the derivative with respect to β of the individual's feasible log-likelihood, unfeasible log-likelihood and true log-likelihood respectively (the score). Define similarly the second-order derivative.

From a standard second-order Taylor expansion of the semiparametric log-likelihood around $\beta_{1,y}$, we have that

$$\sqrt{N}(\hat{\beta}_{1,y}(\hat{P}) - \beta_{1,y}) = \left[\frac{1}{N}\sum_i Ln(\bar{\beta}_{1,y}, \hat{P}_i)_{\beta,\beta}\right]^{-1} \sqrt{N}\frac{1}{N}\sum_i Ln(\beta_{1,y}, \hat{P}_i)_\beta, \qquad (\text{B.3})$$

where $\bar{\beta}_{1,y}$ is between the estimated and true values. By the first part of Assumption B.3 and the consistency of $\hat{\beta}_{1,y}(\hat{P})$, we know that,

$$\left[\frac{1}{N}\sum_i Ln(\bar{\beta}_{1,y}, \hat{P}_i)_{\beta,\beta}\right]^{-1} \xrightarrow{p} \mathbb{E}\left[L(\beta_{1,y}, P_i)_{\beta,\beta}\right]^{-1} =: \Sigma.$$

Now, we focus on the last term in (B.3):

$$\sum_i Ln(\beta_{1,y}, \hat{P}_i)_\beta = \sum_i W_{y,1,i}\frac{\partial log[\Lambda(\beta_{1,y}\hat{H}_i)]}{\partial\beta} + (1 - W_{y,1,i})\frac{\partial log[1 - \Lambda(\beta_{1,y}\hat{H}_i)]}{\partial\beta}$$

$$= \sum_i W_{y,1,i}\begin{bmatrix} \frac{\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)} \\ \frac{\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}C_i \\ \frac{\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}\hat{P}_i \\ \frac{\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}X_i \end{bmatrix}$$

$$
+ \quad (1 - W_{y,1,i}) \begin{bmatrix} \frac{-\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{1-\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)} \\ \frac{-\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{1-\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}C_i \\ \frac{-\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{1-\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}\hat{P}_i \\ \frac{-\Lambda'(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}{1-\Lambda(\beta_0(1,y)+\beta_C(1,y)C_i+\beta_P(1,y)\hat{P}_i+\beta_X(1,y)X_i)}X_i \end{bmatrix}
$$

Considering the path $P_e = (1-e)P + e[\hat{P} - P]$, we take the path-wise derivative of $\sum_i Ln(\beta_{1,y}, P_i)_\beta$ at direction $\hat{P} - P$ (the derivative of the submodel P_e evaluated at $e = 0$). This object is denoted by $\sum_i Ln(\beta_{1,y}, P_i)_{\beta,P_i}$ and is equal to

$$
\begin{aligned}
\sum_i Ln(\beta_{1,y}, P_i)_{\beta,P_i} &= \sum_i W_{y,1,i} \begin{bmatrix} \frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2}\beta_P(1,y)[\hat{P}_i-P_i] \\ \frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2}C_i\beta_P(1,y)[\hat{P}_i-P_i] \\ \left[\frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2}P_i\beta_P(1,y) + \frac{\Lambda'(\beta_{1,y}H_i)}{\Lambda(\beta_{1,y}H_i)}\right][\hat{P}_i-P_i] \\ \frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2}X_i\beta_P(1,y)[\hat{P}_i-P_i] \end{bmatrix} \\[2mm]
&+ \quad \sum_i (1-W_{y,1,i}) \begin{bmatrix} \frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2}\beta_P(1,y)[\hat{P}_i-P_i] \\ \frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2}C_i\beta_P(1,y)[\hat{P}_i-P_i] \\ \left[\frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2}P_i\beta_P(1,y) + \frac{-\Lambda'(\beta_{1,y}H_i)}{1-\Lambda(\beta_{1,y}H_i)}\right][\hat{P}_i-P_i] \\ \frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2}X_i\beta_P(1,y)[\hat{P}_i-P_i] \end{bmatrix}
\end{aligned} \tag{B.4}
$$

We also define $\mathbb{E}\left[Ln(\beta_{1,y}, P_i)_{\beta,P_i}\right]$ analogously.

With these results in hand, we go back to (B.3) and expand around the deviations of the true first stage:

$$
\sqrt{N}(\hat{\beta}_{1,y}(\hat{P}) - \beta_{1,y}) = \Sigma \cdot \sqrt{N}\left(\frac{1}{N}\sum_i Ln(\beta_{1,y}, P_i)_\beta + \frac{1}{N}\sum_i Ln(\beta_{1,y}, P_i)_{\beta,P_i}\right) + o_p(1) \quad \text{(B.5)}
$$

where $\sum_i \frac{1}{N}Ln(\beta_{1,y}, P_i)_\beta$ is the usual estimate of the score, which has mean 0. Thus, if we can show that the second term also has mean 0, the asymptotic normality of our semiparametric estimator follows by a standard multivariate CLT for the vector $\left[\frac{1}{N}\sum_i Ln(\beta_{1,y}, P_i)_\beta, \frac{1}{N}\sum_i Ln(\beta_{1,y}, P_i)_{\beta,P_i}\right]$.

Since all the components of (B.4) have a similar structure, we can focus on one of them and the results are symmetric for the rest.

Consider $\frac{1}{N}\sum_i W_{y,1,i}\frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2}\beta_P(1,y)[\hat{P}_i-P_i]$.

For notation simplicity, let $\frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2}\beta_P(1,y) =: A(\beta_{1,y}H_i)$.

Note that

$$
\frac{1}{N}\sum_i W_{y,1,i}A(\beta_{1,y}H_i)[\hat{P}_i - P_i]
$$

$$
= \frac{1}{N^2}\sum_i\sum_j w_n(Z_i, C_i, Z_j, C_j)W_{y,1,i}A(\beta_{1,y}H_i)\phi_j + o_p(N^{-\frac{1}{2}})
$$

$$
= \frac{1}{N}\sum_i \mathbb{E}\left[w_n(Z_i, C_i, Z, C)\mathbb{E}\left[W_{y,1,i}A(\beta_{1,y}H_i)|H,Z,C\right]|Z_i,C_i\right]\phi_i + o_p(N^{-\frac{1}{2}})
$$

where the first equality is due to Assumption B.3 and the second equality is due to the U-

statistics Hajek projection.

Now, by a standard law of large numbers, we have that

$$\frac{1}{N}\sum_i \mathbb{E}\left[w_n(Z_i, C_i, Z, C)\mathbb{E}\left[W_{y,1,i}A(\beta_{1,y}H_i)|H, Z, C\right]|Z_i, C_i\right]\phi_i + o_p(N^{-\frac{1}{2}})$$

$$\xrightarrow{p} \mathbb{E}[\mathbb{E}\left[w_n(Z_i, C_i, Z, C)\mathbb{E}\left[W_{y,1,i}A(\beta_{1,y}H_i)|H, Z, C\right]|Z_i, C_i\right]\phi_i]$$

$$= \mathbb{E}[\mathbb{E}[\mathbb{E}\left[w_n(Z_i, C_i, Z, C)\mathbb{E}\left[W_{y,1,i}A(\beta_{1,y}H_i)|H, Z, C\right]|Z_i, C_i\right]\phi_i|Z_i, C_i]]$$

$$= \mathbb{E}[\mathbb{E}\left[w_n(Z_i, C_i, Z, C)\mathbb{E}\left[W_{y,1,i}A(\beta_{1,y}H_i)|H, Z, C\right]|Z_i, C_i\right]\mathbb{E}[\phi_i|Z_i, C_i]]$$

$$= 0$$

where the last equality is due to Assumption B.3. Thus, a standard CLT assures the asymptotic normality of the estimator for the parametric part.

B.3.3 Asymptotic behaviour of the DMTE estimator

We derive the influence function of our estimator to be able to express the variance and to apply functional limit theory results. Since DMTE is the difference of two $DMTR$ functions, we can focus on the influence function of one of the $DMTR$ and then apply linearity to get the influence function of the estimator for the DMTE. We will express the $DMTR$ as an unconditional moment that depends on two parameters $(\beta_{d,y}, P)$ that are themselves expressed as unconditional moments.

In particular, P is such that $\mathbb{E}(D|Z, C, X) = P$. Omitting X for simplicity, we can express the moment that determines P with some loss of efficiency as:

$$\mathbb{E}[CZ(D - P)] = 0 \tag{B.6}$$

Similarly, recall that $L(\beta_{1,y}) = \mathbb{E}\left[Ln(\beta_{1,y})\right] = \mathbb{E}\left[W_{y,1,i}log(\Lambda(\beta_{1,y}H)) + (1 - W_{y,1,i})log(1 - \Lambda(\beta_{1,y}H))\right]$ is a non-random function that is continuous and differentiable in $\beta_{1,y}$. Note $L(\beta_{1,y})$ is itself a function of P since P is inside H. We can rewrite this as a moment equality using the score as:

$$\mathbb{E}[S(\beta_{1,y}[P])] = \mathbb{E}\left[W_{y,1,i}\frac{\partial log(\Lambda(\beta_{1,y}H))}{\partial \beta_{1,y}} + (1 - W_{y,1,i})\frac{\partial log(1 - \Lambda(\beta_{1,y}H))}{\partial \beta_{1,y}}\right] = 0 \tag{B.7}$$

Let $N = (P, \mathbf{1}\{C > y\})$. Then, the moment that identifies the $DMTR_1$ is: $DMTR_1 = \mathbb{E}[\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)\hat{P})\beta_P(1, y)|N]$, or, using an unconditional representation with some efficiency loss,

$$\mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1) \cdot P \cdot \mathbf{1}\{C > y\}] = 0 \tag{B.8}$$

To be more specific, the relevant moments that define the parameters of interest are

$$0 = \mathbb{E}[CZ(D - P)] \tag{B.9}$$

$$0 = \mathbb{E}\left[W_{y,1}\frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)}{\Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)}\right.$$

$$-(1 - W_{y,1})\frac{\Lambda'((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}{1 - \Lambda((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}\Bigg] \tag{B.10}$$

$$
0 = \mathbb{E}\Bigg[W_{y,1}\frac{\Lambda'((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}{\Lambda((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}C \\
-(1 - W_{y,1})\frac{\Lambda'((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}{1 - \Lambda((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}C\Bigg] \tag{B.11}
$$

$$
0 = \mathbb{E}\Bigg[W_{y,1}\frac{\Lambda'((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}{\Lambda((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}P \\
-(1 - W_{y,1})\frac{\Lambda'((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}{1 - \Lambda((\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)}P\Bigg] \tag{B.12}
$$

$$0 = \mathbb{E}\left[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y) - DMTR_1)\cdot P \cdot \mathbf{1}\{C > y\}\right] \tag{B.13}$$

We follow Newey (1994), Ichimura and Newey (2022), and Ackerberg, Chen, Hahn and Liao (2014). We will assume that standard conditions for the interchange of integration and differentiation hold (such as dominated convergence theorem conditions).

Before proceeding, note that:

$$\mathbb{E}[CZ(D - P)] = \int_{C \times Z \times D} cz(d - P)f(c,z,d)d\mu(c,z,d) = \int_{\mathcal{D}} cz(d - P)f(\delta)d\mu(\delta) \tag{B.14}$$

This also holds for the rest of the moments, where δ is the full data vector. This is useful to be able to replicate the form of Equation (3.10) in Newey (1994), which is instrumental in deriving the influence functions.

We want to derive the influence function of $DMTR_1$. Thus, we can start with (B.13) and consider a parametric submodel t for the nuisance parameters and differentiate. Let $s(\delta)$ be the score of the data.

$$
\frac{\partial \mathbb{E}_t[(\Lambda'(\beta_0\,(1,y)\,(t) + \beta_C\,(1,y)\,(t)C + \beta_P\,(1,y)\,P(t))\beta_P\,(1,y)\,(t) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t}\Bigg|_{t=0}
$$

$$
= \mathbb{E}[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y) - DMTR_1)P\mathbf{1}\{C > y\}\,s(\delta)]
$$
$$
+ \frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y)\,(t) + \beta_C\,(1,y)\,(t)C + \beta_P\,(1,y)\,(t)P)\beta_P\,(1,y)\,(t) - DMTR_1)P\mathbf{1}\{C > y\}]}{\partial t}\Bigg|_{t=0}
$$
$$
+ \frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P(t))\beta_P\,(1,y) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t}\Bigg|_{t=0}
$$

By the implicit function theorem, we have that

$$
\frac{\partial DMTR_1}{\partial t}\Bigg|_{t=0}
$$
$$
= [-\mathbb{E}[P\mathbf{1}\{C > y\}]]^{-1}\Big[\mathbb{E}[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y) - DMTR_1)P\mathbf{1}\{C > y\}\,s(\delta)]
$$

$$+ \left.\frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y)\,(t) + \beta_C\,(1,y)\,(t)C + \beta_P\,(1,y)\,(t)P)\beta_P\,(1,y)\,(t) - DMTR_1)P\mathbf{1}\,\{C > y\}]}{\partial t}\right|_{t=0}$$

$$+ \left.\frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P(t))\beta_P\,(1,y) - DMTR_1)P(t)\mathbf{1}\,\{C > y\}]}{\partial t}\right|_{t=0}.$$

Now, we need to express the second two components as products with the score of the data to apply Equation (3.10) of Newey (1994). We start with

$$\frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y)\,(t) + \beta_C\,(1,y)\,(t)C + \beta_P\,(1,y)\,(t)P)\beta_P\,(1,y)\,(t) - DMTR_1)P\mathbf{1}\,\{C > y\}]}{\partial t},$$

which is the parametric part of the model. We have that

$$\left.\frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y)\,(t) + \beta_C\,(1,y)\,(t)C + \beta_P\,(1,y)\,(t)P)\beta_P\,(1,y)\,(t) - DMTR_1)P\mathbf{1}\,\{C > y\}]}{\partial t}\right|_{t=0}$$

$$= \mathbb{E}\left[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P) \cdot \beta_P\,(1,y) \cdot P \cdot \mathbf{1}\,\{C > y\}\right.$$

$$\left. \cdot \left(\left.\frac{\partial \beta_0\,(1,y)\,(t)}{\partial t}\right|_{t=0} + \left.\frac{\partial \beta_C\,(1,y)\,(t)}{\partial t}\right|_{t=0} \cdot C + \left.\frac{\partial \beta_P\,(1,y)\,(t)}{\partial t}\right|_{t=0} \cdot P\right)\right]$$

We claim, and will become evident further on that is indeed true, that, using Equations (B.9)-(B.12), we can express $\frac{\partial \beta_A(1,y)(t)}{\partial t} = \mathbb{E}[IF_{\beta_A(1,y)}s(\delta)]$, where $IF_{\beta_A(1,y)}$ is the influence function of $\beta_A\,(1,y)$ and A is a random variable.

Then, we have that

$$\left.\frac{\partial DMTR_1}{\partial t}\right|_{t=0}$$

$$= [-\mathbb{E}[P\mathbf{1}\,\{C > y\}]]^{-1}$$

$$\cdot \left[E\big[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y) - DMTR_1)P\mathbf{1}\,\{C > y\}\,s(\delta)\big]\right.$$

$$+ E\big[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y)\,P\mathbf{1}\,\{C > y\}$$

$$\cdot (\mathbb{E}[IF_{\beta_0(1,y)}s(\delta)] + \mathbb{E}[IF_{\beta_C(1,y)}s(\delta)]C + \mathbb{E}[IF_{\beta_P(1,y)}s(\delta)]P)\big]$$

$$\left.+ \left.\frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P(t))\beta_P\,(1,y) - DMTR_1)P(t)\mathbf{1}\,\{C > y\}]}{\partial t}\right|_{t=0}\right],$$

or, equivalently,

$$\left.\frac{\partial DMTR_1}{\partial t}\right|_{t=0}$$

$$= [-\mathbb{E}[P\mathbf{1}\,\{C > y\}]]^{-1}\Big[E\big[\{(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y) - DMTR_1)P\mathbf{1}\,\{C > y\}$$

$$+ \mathbb{E}[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y))P\mathbf{1}\,\{C > y\}]IF_{\beta_0(1,y)}$$

$$+ \mathbb{E}[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y))P\mathbf{1}\,\{C > y\})C]IF_{\beta_C(1,y)}$$

$$+ \mathbb{E}[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y))P\mathbf{1}\,\{C > y\})P]IF_{\beta_P(1,y)}\}s(\delta)\big]$$

$$+ \left.\frac{\partial \mathbb{E}[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P(t))\beta_P\,(1,y) - DMTR_1)P(t)\mathbf{1}\,\{C > y\}]}{\partial t}\right|_{t=0}\Big].$$

Now, we need to derive the non-parametric component

$$\left.\frac{\partial\mathbb{E}[(\Lambda'(\beta_0\,(1,y)+\beta_C\,(1,y)\,C+\beta_P\,(1,y)\,P(t))\beta_P\,(1,y)-DMTR_1)P(t)\mathbf{1}\,\{C>y\}]}{\partial t}\right|_{t=0}.$$

We assume that the conditions for the Riesz Representation Theorem hold for

$$\frac{\partial\mathbb{E}[(\Lambda'(\beta_0\,(1,y)+\beta_C\,(1,y)\,C+\beta_P\,(1,y)\,P(t))\beta_P\,(1,y)-DMTR_1)P(t)\mathbf{1}\,\{C>y\}]}{\partial t}.$$

See, for example, Ackerberg et al. (2014) for such conditions as being a linear bounded functional. Then, there is a unique b in a properly defined space with the inner product $<b_1,b_2>=\mathbb{E}[b_1b_2]$ such that:

$$\frac{\partial\mathbb{E}[(\Lambda'(\beta_0\,(1,y)+\beta_C\,(1,y)\,C+\beta_P\,(1,y)\,P(t))\beta_P\,(1,y)-DMTR_1)P(t)\mathbf{1}\,\{C>y\}]}{\partial t}$$

$$= \mathbb{E}\left[b\frac{\partial\,\{CZ(D-P(t))\}}{\partial t}\right]$$

$$= \frac{\partial\mathbb{E}[bCZ(D-P(t))]}{\partial t}$$

$$= \mathbb{E}\left[\tilde{b}(\delta)\frac{\partial P(t)}{\partial t}\right]$$

$$= \frac{\partial\mathbb{E}[\tilde{b}(\delta)P(t)]}{\partial t} \tag{B.15}$$

where $\tilde{b}(\delta)=-bCZ$.

To find the *IF* following Ichimura and Newey (2022), we need to find a $\phi(\delta,P,\alpha)$ such that:

$$\frac{\partial\mathbb{E}[(\Lambda'(\beta_0\,(1,y)+\beta_C\,(1,y)\,C+\beta_P\,(1,y)\,P(t))\beta_P\,(1,y)-DMTR_1)P(t)\mathbf{1}\,\{C>y\}]}{\partial t}$$

$$= \int\phi(\delta,P,\alpha)G(d\delta) \tag{B.16}$$

where G is a perturbation from the true *CDF*.

Furthermore, from (B.9), we can see that

$$0 \;=\; \frac{\partial\mathbb{E}_t[CZ(D-P(t))]}{\partial t}=\int CZ(D-P)G(d\delta)+\frac{\partial\mathbb{E}[CZ(D-P(t))]}{\partial t}$$

or, equivalently,

$$\int CZ(D-P)G(d\delta)=-\frac{\partial\mathbb{E}[CZ(D-P(t))]}{\partial t}.$$

Thus, following Ichimura and Newey (2022), if we find an $\alpha(\delta)$ such that:

$$\frac{\partial\mathbb{E}[(\Lambda'(\beta_0\,(1,y)+\beta_C\,(1,y)\,C+\beta_P\,(1,y)\,P(t))\beta_P\,(1,y)-DMTR_1)P(t)\mathbf{1}\,\{C>y\}]}{\partial t}$$

$$= -\frac{\partial\mathbb{E}[\alpha(\delta)CZ(D-P(t))]}{\partial t}\partial t \tag{B.17}$$

then we will get (B.16).

Furthermore, from (B.9), we can see that

$$\frac{\partial\mathbb{E}[CZ(D-P(t))]}{\partial t}=\mathbb{E}\left[\frac{\partial CZ(D-P(t))}{\partial t}\right]=\mathbb{E}\left[a(\delta)\frac{\partial P(t))}{\partial t}\right]=\frac{\partial\mathbb{E}[a(\delta)P(t)]}{\partial t} \tag{B.18}$$

where $a(\delta) = CZ$.

From (B.15) and (B.18) combined with (B.17):

$$\frac{\partial \mathbb{E}[\tilde{b}(\delta)P(t)]}{\partial t} = -\frac{\partial \mathbb{E}[\alpha(\delta)a(\delta)P(t)]}{\partial t} \tag{B.19}$$

The equality in Equation (B.19) will be satisfied if it holds for every t. Since $P(t)$ is in the space of possible propensity scores, the condition will be satisfied if it holds for any P in the space of possible propensity scores. Thus:

$$\mathbb{E}[\tilde{b}(\delta)P] = -\mathbb{E}[\alpha(\delta)a(\delta)P] \tag{B.20}$$

or, equivalently,

$$0 = \mathbb{E}[\{\tilde{b}(\delta) + \alpha(\delta)a(\delta)\}P] = \mathbb{E}\left[(-a(\delta))\left\{\frac{-\tilde{b}(\delta)}{a(\delta)} - \alpha(\delta)\right\}P\right] \tag{B.21}$$

Thus, as in Ichimura and Newey (2022), the $\alpha(\delta)$ that minimizes $\mathbb{E}\left[(-a(\delta))\left\{\frac{-\tilde{b}(\delta)}{a(\delta)} - \alpha(\delta)\right\}^2\right]$ satisfies

$$\phi(\delta, P, \alpha) = \alpha(\delta)CZ(D - P).$$

Combining this result with the previous display, the Influence Function of $DMTR_1$ is given by

$$IF_{DMTR_1}$$
$$= [-\mathbb{E}[P\mathbf{1}\{C > y\}]]^{-1}\Big[(\Lambda'(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y) - DMTR_1)P\mathbf{1}\{C > y\}$$
$$+ \mathbb{E}[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y))P\mathbf{1}\{C > y\})]IF_{\beta_0(1,y)}$$
$$+ \mathbb{E}[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y))P\mathbf{1}\{C > y\})C]IF_{\beta_C(1,y)}$$
$$+ \mathbb{E}[\Lambda''(\beta_0\,(1,y) + \beta_C\,(1,y)\,C + \beta_P\,(1,y)\,P)\beta_P\,(1,y))P\mathbf{1}\{C > y\})P]IF_{\beta_P(1,y)}$$
$$+ \phi(\delta, P, \alpha)\Big] \tag{B.22}$$

The influence function will depend on the influence functions of the estimated coefficients. Their influence function could be derived similarly to the nonparametric effect of P on $DMTR_1$, but using Equations (B.9)-(B.12). We could then use a similar logic as we did following Ichimura and Newey (2022) and a form of Riezs representation. That would fully complete the representation, implying that our previous claim holds.

Since $DMTE = DMTR_1 - DMTR_0$, we can then say that:

$$IF_{DMTE} = IF_{DMTR_1} - IF_{DMTR_0} \tag{B.23}$$

where IF_{DMTR_0} can be derived analogously to IF_{DMTR_1}.

Then, we know that

$$\sqrt{N}(\widehat{DMTE} - DMTE) = \frac{1}{\sqrt{n}}\sum_i^n IF_{DMTE}(\delta_i) + o_p(1) \xrightarrow{d} N(0, Var(DMTE))$$

and

$$Var(DMTE) = \mathbb{E}[IF_{DMTE}^2],$$

where convergence is due to standard results on influence functions (van der Vaart and Wellner, 1996; van der Vaart, 1998).

Following Frandsen (2015) and our influence function calculations, we can recover the asymptotic distribution of the $QMTE[\tau, p]$. The $QMTR_d$ as quantiles have the following standard and known influence functions:

$$IF_{QMTR_d(\tau,p)} = -\frac{1\{DMTR_d^{-1}(\tau,p) > y\} - \tau}{\frac{\partial DMTR_d(DMTR_d^{-1}(\tau,p),p)}{\partial y}},$$

implying that $IF_{QMTE(\tau,p)} = IF_{QMTR_1(\tau,p)} - IF_{QMTR_0(\tau,p)}$,

$$\sqrt{N}(\widehat{QMTE} - QMTE) = \frac{1}{\sqrt{n}}\sum_i^n IF_{QMTE}(\delta_i) + o_p(1) \xrightarrow{d} N(0, Var(QMTE))$$

and

$$Var(QMTE) = \mathbb{E}[IF_{QMTE}^2],$$

where convergence is due to standard results on influence functions (van der Vaart and Wellner, 1996; van der Vaart, 1998).

By recalling that $MTE(p) = \int_0^1 QMTE(\tau, p)d\tau$ and the fact that we just provided asymptotic normality for $QMTE(\tau, p)$ we can recover the distribution of the $MTE(p)$ following Masten, Poirier and Zhang (2020).

At this point, it is worth being specific about the definition of Hadamard differentiability and how it connects to the QTE and the MTE.

Definition 1. *Let $\phi : D \to E$ where D, E are Banach spaces. Say ϕ is Hadamard differentiable at $\theta \in D$ if $\exists \phi'_\theta : D \to E$, $\forall h \in D$, if $t \to 0$, $||h_t - h|| \to 0$, then:*

$$\left|\left|\frac{\phi(\theta + th_t) - \phi(\theta)}{t} - \phi'_\theta(h)\right|\right|_E \to 0$$

In our context, we set $D = \mathcal{C}([0,1],[0,1])$ and $E = \mathcal{R}$, i.e., D is the space of continuous functions where the first component refers to τ and the second one to v. Then, we know that $\frac{QMTE(\theta_1 + th_{1t}, \theta_2 + th_{2t}) - QMTE(\theta_1, \theta_2)}{t} \xrightarrow{||||_R} QMTE'_{\theta_1,\theta_2}(h_1, h_2)$ and $\frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} \xrightarrow{||||_R}$ $QMTE'_{\theta_2}(h_2)$, where $||||_R$ denotes the norm of convergence. Furthermore, we have that

$$\frac{MTE(\theta_2 + th_{2t}) - MTE(\theta_2)}{t} = \int_0^1 \frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t}d\tau,$$

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which, under the conditions for the dominated convergence theorem, implies that

$$\frac{MTE(\theta_2 + th_{2t}) - MTE(\theta_2)}{t} = \int_0^1 \frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} d\tau$$

$$\rightarrow \int_0^1 QMTE'_{\theta_2}(h_2) d\tau := MTE'_{\theta_2}(h_2).$$

Consequently, the MTE is Hadamard differentiable, and we can apply the functional delta method again to get the asymptotic Gaussian distribution of the MTE.

Furthermore, by the chain rule of influence functions,

$$IF_{MTE(p)} = \int_0^1 IF_{QMTE(\tau, p)} d\tau$$

Although relevant to show the results for the MTE, in our text, we also focus on the $RMTE(v)$ to avoid additional support assumptions. Thus, we can similarly derive the asymptotic results for the RMTE, since $RMTE(v) = -\int_0^{\gamma_C} DMTE(y, v) \, dy$. In the context of the RMTE, we set $D = \mathcal{C}([0, \gamma], [0, 1])$ and $E = \mathcal{R}$, i.e., D is the space of continuous functions where the first component refers to y, $\gamma = \min\{\gamma_1, \gamma_0\}$ and the second one to v. Then, we know that $\frac{DMTE(\theta_1 + th_{1t}, \theta_2 + th_{2t}) - DMTE(\theta_1, \theta_2)}{t} \longrightarrow_{||||_R} DMTE'_{\theta_1, \theta_2}(h_1, h_2)$ and $\frac{DMTE(\tau, \theta_2 + th_{2t}) - DMTE(y, \theta_2)}{t} \longrightarrow_{||||_R} DMTE'_{\theta_2}(h_2)$, where $||||_R$ denotes the norm of convergence. Furthermore, we have that

$$\frac{RMTE(\theta_2 + th_{2t}) - RMTE(\theta_2)}{t} = -\int_0^{\gamma_C} \frac{DMTE(y, \theta_2 + th_{2t}) - DMTE(y, \theta_2)}{t} dy,$$

which, under the conditions for the dominated convergence theorem, implies that

$$\frac{RMTE(\theta_2 + th_{2t}) - RMTE(\theta_2)}{t} = -\int_0^{\gamma_C} \frac{DMTE(y, \theta_2 + th_{2t}) - DMTE(y, \theta_2)}{t} dy$$

$$\rightarrow -\int_0^{\gamma_C} DMTE'_{\theta_2}(h_2) dy := RMTE'_{\theta_2}(h_2).$$

Consequently, the RMTE is Hadamard differentiable, and we can apply the functional delta method to get the asymptotic Gaussian distribution.

Furthermore, by the chain rule of influence functions,

$$IF_{RMTE(p)} = -\int_0^{\gamma_C} IF_{DMTE(y, p)} dy.$$

B.4 Proof of Theorems 4.2 and H.2: Validity of the Weighted Bootstrap

We generate $\{V_i, i = 1, \ldots, n\}$ as a sequence of independent and identically distributed non-negative random variables with mean one, variance one, and finite third moment (e.g., $V_i \sim Exp(1)$).

Based on this we estimate the propensity score by minimizing the weighted version of the standard series minimization criteria,

$$\hat{\theta}^{fs,*} = \underset{\theta^{fs} \in \Theta^{fs}}{\arg\min} \, n^{-1} \sum_{i=1}^{n} V_i \left(D_i - \alpha_0 - X_i'\alpha_X - C_i\alpha_C - \psi^L(Z_i)'\alpha_Z, \right)^2 \quad \text{(B.24)}$$

where $\hat{\theta}^{fs,*} = (\hat{\alpha}_0^*, \hat{\alpha}_X^{*\prime}, \hat{\alpha}_C^*, \hat{\alpha}_Z^*))'$ and thus $\hat{P}_i^*$ is a function of $\hat{\theta}^{fs,*}$. Note then that by Corollary 3 in Ma and Kosorok (2005a) or Corollary 3.2.3 and Theorem 3.2.5 of van der Vaart and Wellner (1996), $\hat{P}_i^*$ converges to P_i.

For the estimation of the βs, let:

$$Ln^*(\beta_{1,y}, \hat{P}) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i V_i W_{y,1,i} log[\Lambda(\beta_{1,y}\hat{H}_i)] + V_i(1 - W_{y,1,i}) log[1 - \Lambda(\beta_{1,y}\hat{H}_i)] \quad \text{(B.25)}$$

And,

$$Ln^*(\beta_{1,y}, P) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i V_i W_{y,1,i} log[\Lambda(\beta_{1,y}H_i)] + V_i(1 - W_{y,1,i}) log[1 - \Lambda(\beta_{1,y}H_i)] \quad \text{(B.26)}$$

By a similar display as in the unweighted case, we know that:

$$\sup_{\beta_{1,y}} |Ln^*(\beta_{1,y}, \hat{P}) - Ln^*(\beta_{1,y}, P)| = o_p(1)$$

Where $Ln^*(\beta_{1,y}, P)$ is a standard, but weighted parametric likelihood and thus by a second application of Corollary 3 in Ma and Kosorok (2005a) or Corollary 3.2.3 and Theorem 3.2.5 of van der Vaart and Wellner (1996), $\hat{\beta}_{1,y}^*$ converges to $\beta_{1,y}$.

It then remains to show asymptotic normality of $\hat{\beta}_{1,y}^*$ and then of $DMTR^*$. Since the V_i are independent of everything and $Var(V_i) = 1$, we have $\sqrt{N}(\hat{\beta}_{1,y}^* - \beta_{1,y})$ is asymptotically normal as long as a version of Assumption B.3 holds with $\hat{P}_i^*$.

Furthermore, since V_i is independent of everything, similar calculations of the influence functions for $DMTE, QMTE,$ and $RMTE$ can be provided to obtain asymptotic normality of the bootstrapped versions of these. Alternatively, one can note that $DMTE, QMTE,$ and $RMTE$ are all continuous and Hadamard differentiable functions of $\beta_{1,y}, P_i$, with $\hat{P}^*$ slower than root-n via a bootstrap version of Assumption B.3 and thus functional delta methods and continuous mapping theorems hold.

C   Constructing the Dataset

In this appendix, we summarize Appendix I.2 of Possebom (2023), which provides a detailed explanation of how the dataset used in our empirical application was constructed. We explain the specific crime types included in our sample, the classification algorithms used to define which defendants were punished, and the fuzzy matching algorithm used to define which defendants recidivate. At the end, we explain how we constructed the extra covariates used to assess the validity of our identifying assumptions.

The final dataset was created from three initial datasets.

1. CPOPG ("Consulta de Processos de Primeiro Grau"): It contains information about all criminal cases in the Justice Court System in the State of São Paulo (TJ-SP) between 2010 and 2019.

2. CJPG ("Consulta de Julgados de Primeiro Grau"): It contains information about the last decision made by a trial judge in all criminal cases in TJ-SP between 2010 and 2019.

3. CPOSG ("Consulta de Processos de Segundo Grau"): It contains information about all appealing criminal cases in TJ-SP between 2010 and 2019.

Starting from the CPOPG dataset, we implement the following steps.

1. We only keep cases that are currently in the Appeals Court, closed, or whose status is empty. Those cases are already associated with a trial judge's sentence.

2. We only keep cases whose crime types are associated with sentences that must be less than four years of incarceration.

3. We only keep cases that aim to analyze whether a defendant is guilty or not.

4. We only keep cases that were randomly assigned to trial judges.

5. We only keep cases whose starting date is after January 1$^{\text{st}}$, 2010.

After these steps, our dataset contains 98,552 cases. We then merged it with the CJPG dataset using cases' `id` codes. Since some cases do not have `id` codes, our dataset now contains 98,422 cases.

After this step, we randomly select 35 cases per year (2010-2019) for manual classification. We manually classify them into five categories: "defendant died during the trial", "defendant is guilty", "defendant accepted a non-prosecution agreement" ("transação penal" in Portuguese), "case was dismissed" ("processo suspenso" in Portuguese) and "defendant was acquitted". Since some sentences are missing or incomplete, we are able to manually classify only 325 sentences.

Now, we use those 325 manually classified cases to train a classification algorithm. To do so, we divide them into a training sample (216 cases) and a validation sample (109 sentences).

First, we design an algorithm to identify which defendants died during the trial. To do so, we check whether the sentence contains any reference to the first paragraph of Article 107 from the Brazilian Criminal Code. This deterministic algorithm perfectly classifies cases into the category "defendant died during the trial".

Second, we design an algorithm to identify which cases were dismissed. To do so, we check whether the sentence contains any reference to Article 89 in Law n. 9099/95. This deterministic algorithm correctly classifies 98% of the cases into the category "case was dismissed".

Third, we design an algorithm to identify which defendants accepted a non-prosecution agreement. To do so, we check whether the sentence contains any expression connected to a non-prosecution agreement. This deterministic algorithm correctly classified almost all the cases into the category "defendant accepted a non-prosecution agreement", making only three mistakes.

Finally, we design an algorithm to classify the remaining cases into two categories: "defendant is guilty" and "defendant was acquitted". To do so, we define a bag of words that were found to be strong signals of acquittal and guilt when manually classifying the cases in our samples. We then count how many times each one of those expressions appears in each sentence, and we normalize those counts to be between 0 and 1.

Using the normalized counts, we train an L1-Regularized Logistic Regression using our training sample. We then validate this algorithm using our validation sample and find that it correctly classifies 98.8% of the cases. Given this high success rate, we use the L1-Regularized Logistic Regression algorithm to define the treatment variable in our full sample.

Having designed the above algorithm, we use it to define the trial judge's treatment variable T in the full sample. First, we find which defendants died during their trials and drop them from our sample. We then use the second and third algorithms to define which cases were dismissed and which cases are associated with a non-prosecution agreement. Moreover, we use the trained L1-regularized Logistic Regression algorithm to classify the remaining cases into the categories "defendant is guilty" and "defendant was acquitted". Finally, we combine the categories "defendant was acquitted" and "case was dismissed" into the untreated group ("not punished", $T = 0$) and the categories "defendant accepted a non-prosecution agreement" and "defendant is guilty" into the treated group ("punished", $T = 1$). At the end, our dataset contains 96,225 cases.

Now, we merge our current dataset with the CPOSG dataset using each case's `id` code. When merging these datasets, we create an indicator variable that denotes which cases went to the Appeals Court, i.e., which cases were matched. We then randomly select 50 cases per year for manual classification (2010-2019) and divide them into three categories: "cases that went

to the Appeals Court, but were immediately returned due to bureaucratic errors", "cases whose trial judge's sentences were affirmed" and "cases whose trial judge's sentences were reversed".

Now, we use those 500 manually classified cases to train a classification algorithm. To do so, we divide them into a training sample (300 cases) and a validation sample (200 sentences).

First, we design an algorithm to identify which cases went to the Appeals Court but were immediately returned. To do so, we simply check whether the Appeals Court's `decision` is empty.

Finally, we design an algorithm to classify the non-empty cases into two categories: "cases whose trial judge's sentences were affirmed" and "cases whose trial judge's sentences were reversed". To do so, we define a bag of words that were found to be strong signals of sentence reversal when manually classifying the cases in our sample. We then count how many times each one of those expressions appears in each sentence, and we normalize those counts to be between 0 and 1.

Using the normalized counts, we train an L1-Regularized Logistic Regression using our training dataset. We then validate this algorithm using our validation sample and find that it correctly classifies 96.2% of the cases. Given this high success rate, we use the L1-Regularized Logistic Regression to define the treatment variable in our full sample.

Having designed the above algorithms, we use them to define the final treatment variable D in the full sample. First, we set $D = T$ if a case did not go to the Appeals Court or if a case went to the Appeals Court, but was immediately returned. Second, we use the trained L1-Regularized Logistic Regression algorithm to classify the remaining cases into the categories "reversed trial judge's sentence" and "affirmed trial judge's sentence". We, then, set $D = T$ if the trial judge's sentence was affirmed and $D = 1 - T$ if the trial judge's sentence was reversed. Moreover, we also drop the cases whose dates (starting date, trial judge's sentence date and Appeal Court's decision date) are not appropriately ordered. At the end, our dataset contains 95,119 cases.

Now, our goal is to find the defendants' names in each case. To do so, we use the variable `parties` from the CPOPG dataset and search for names listed as defendants. Finally, we delete names that are not a person's name — such as district attorney offices, public defender offices and "unknown author". Our sample now contains 103,423 case-defendant pairs.

Furthermore, we repeat the steps in the last paragraph to find defendants' names in a dataset that contains all cases from the CPOPG dataset, including cases that are still open and cases with severe crimes. This dataset contains 1,027,120 case-defendants pairs.

Now, we use these two datasets to define our outcome variable (Y = "time to recidivism"). A defendant i in a case j in the smaller dataset recidivates if and only if defendant i's full name appears in a case $\bar{j}$ in the larger dataset. To match defendants' names across cases, we use the Jaro–Winkler similarity metric and we define a match if the similarity between full names in

two different cases is greater than or equal to 0.95. If we find a match, we define the outcome variable as the time difference between the second case's start date and the first case's final date.

Furthermore, we delete the case-defendant pairs whose cases started in 2018 and 2019. Consequently, our dataset contains 51,731 case-defendants pairs.

Lastly, we use the CPOPG dataset to create four covariates that are used to assess the validity of our identifying assumptions in Section 5.1. Following Possebom (2023), we use the defendant's name to find whether the defendant has a typically male name according to the Brazilian 2010 Census (*R package* genderBR). Following Laneuville and Possebom (2024), we use each case's events to find which defendants were caught red-handed when committing a crime (*in flagrante delicto*), and we use each case's list of parties to find which defendants used public defenders. Moreover, we also use information on the type of crime to create an indicator variable of whether the defendant was accused of theft.
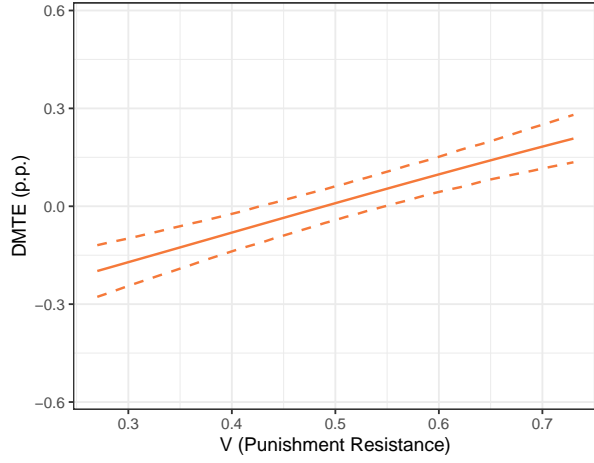
D   Additional Empirical Results

Figure D.1: $QMTE\left(\cdot, v\right)$ for $v \in \{.3, .4, \dots, .7\}$
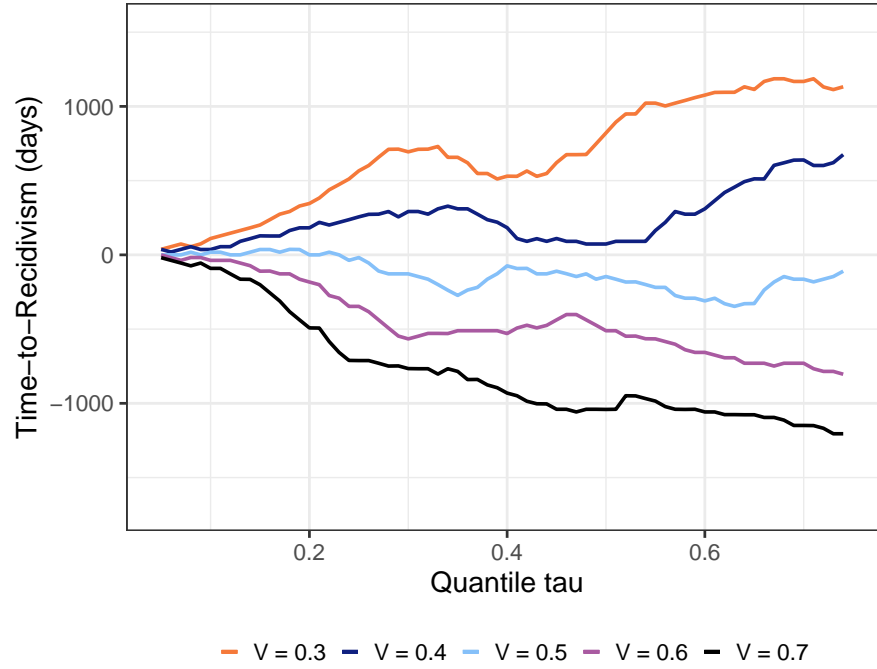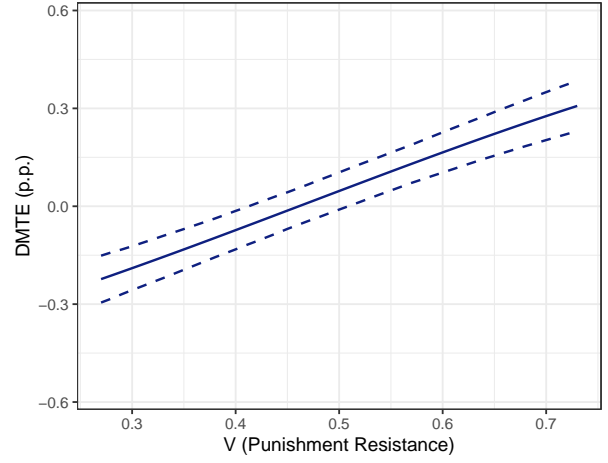

Notes: Solid lines are the point estimates for the average $QMTE\left(\cdot, v\right)$ functions indicated in the legend. These results are based on Corollary 4.1.

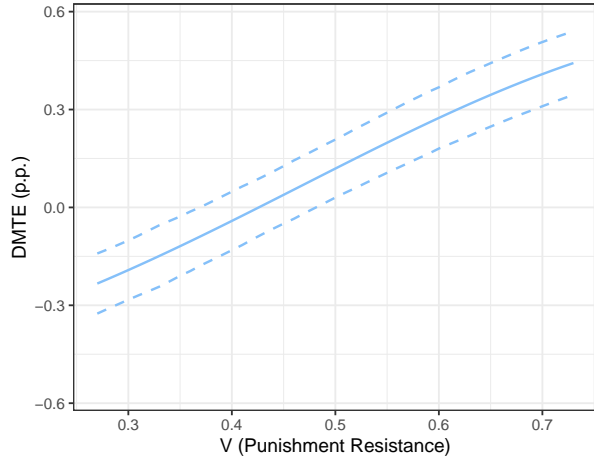## D.1 Confidence Intervals for DMTE, QMTE and RMTE functions

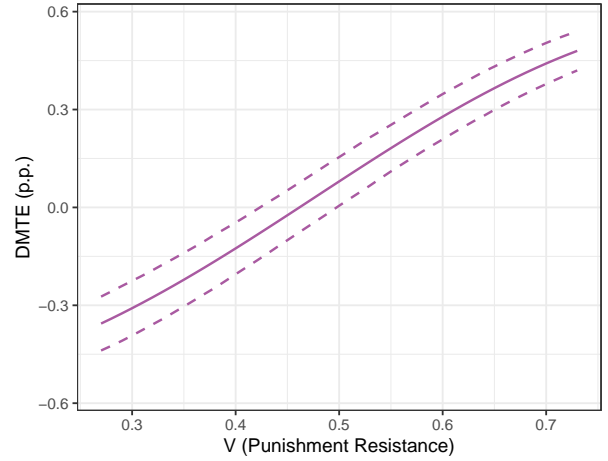Figure D.2: 90%-Confidence Intervals for $DMTE\left(y,\cdot\right)$ for $y\in\left\{1,2,3,4\right\}$

(a) $DMTE\left(1,\cdot\right)$

(b) $DMTE\left(2,\cdot\right)$

(c) $DMTE\left(3,\cdot\right)$

(d) $DMTE\left(4,\cdot\right)$

Notes: Solid lines are the point estimates for the average $DMTE\left(y,\cdot\right)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clusterized at the court district level (Subsection 4.2) using 399 repetitions.

Figure D.3: 90%-Confidence Intervals for $DMTE(y, \cdot)$ for $y \in \{5, 6, 7, 8\}$

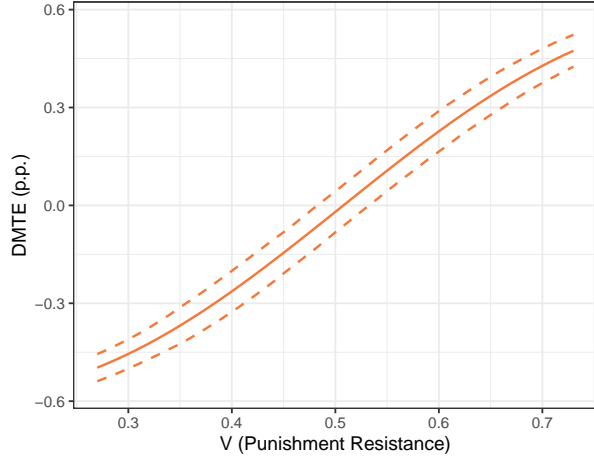(a) $DMTE(5, \cdot)$

(b) $DMTE(6, \cdot)$
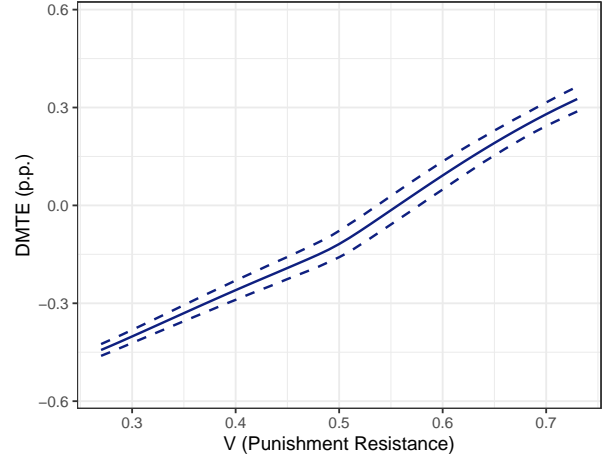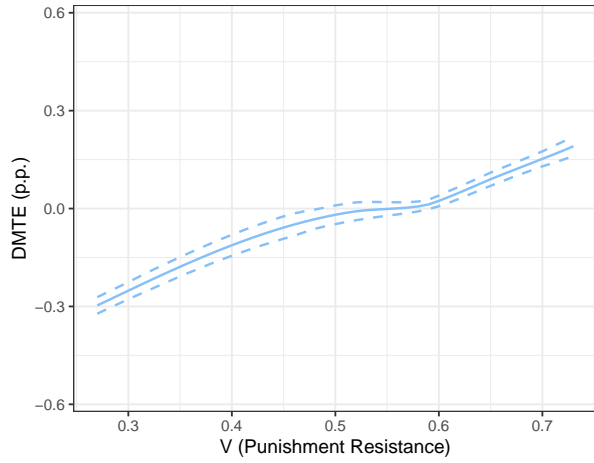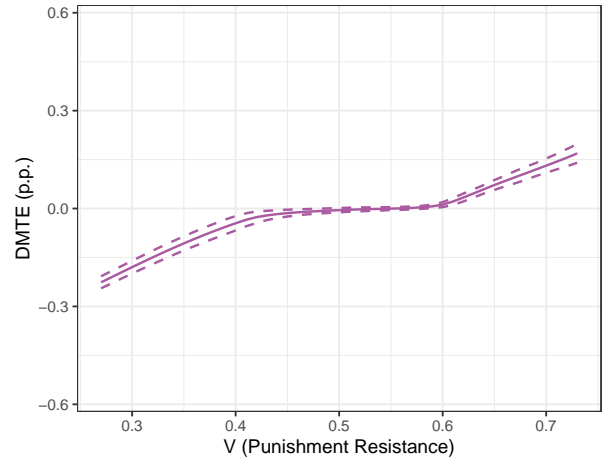
(c) $DMTE(7, \cdot)$

(d) $DMTE(8, \cdot)$

Notes: Solid lines are the point estimates for the average $DMTE(y, \cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clusterized at the court district level (Subsection 4.2) using 399 repetitions.

Figure D.4: 90%-Confidence Intervals for $QMTE(\tau, \cdot)$ for $\tau \in \{.10, .15, .25, .30\}$

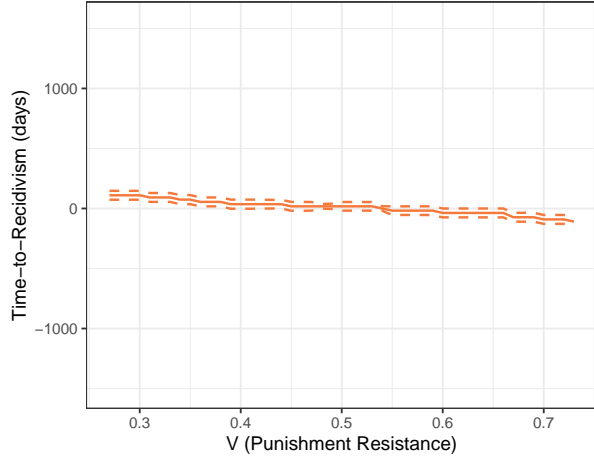(a) $QMTE(.10, \cdot)$
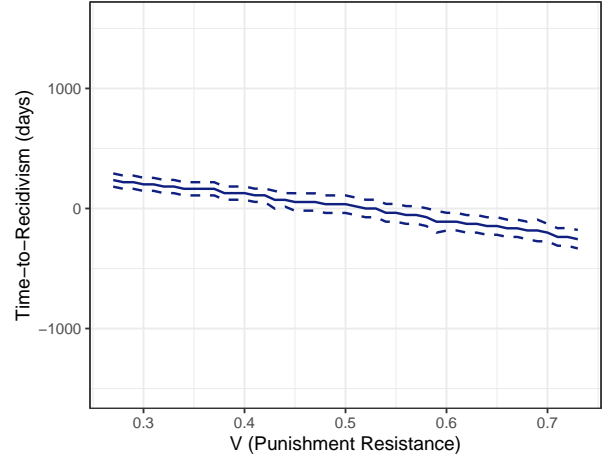
(b) $QMTE(.15, \cdot)$
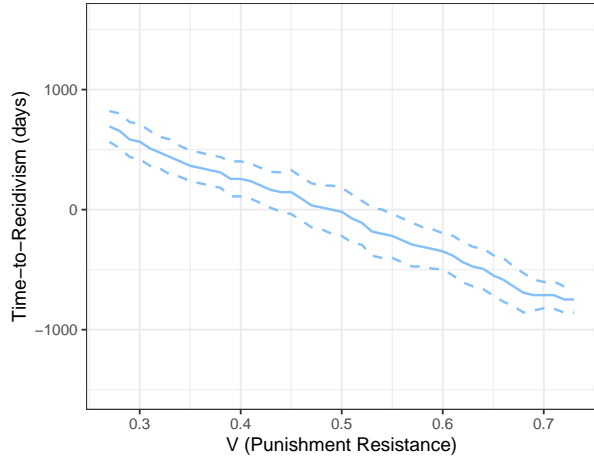
(c) $QMTE(.25, \cdot)$
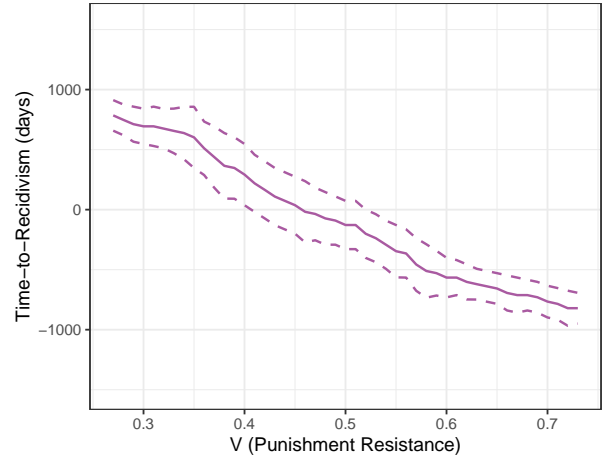
(d) $QMTE(.30, \cdot)$

Notes: Solid lines are the point estimates for the average $QMTE(\tau, \cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clusterized at the court district level (Subsection 4.2) using 399 repetitions.

Figure D.5: 90%-Confidence Intervals for $QMTE(\tau, \cdot)$ for $\tau \in \{.40, .50, .75\}$ and $RMTE(\cdot)$

(a) $QMTE(.40, \cdot)$

(b) $QMTE(50, \cdot)$

(c) $QMTE(.75, \cdot)$
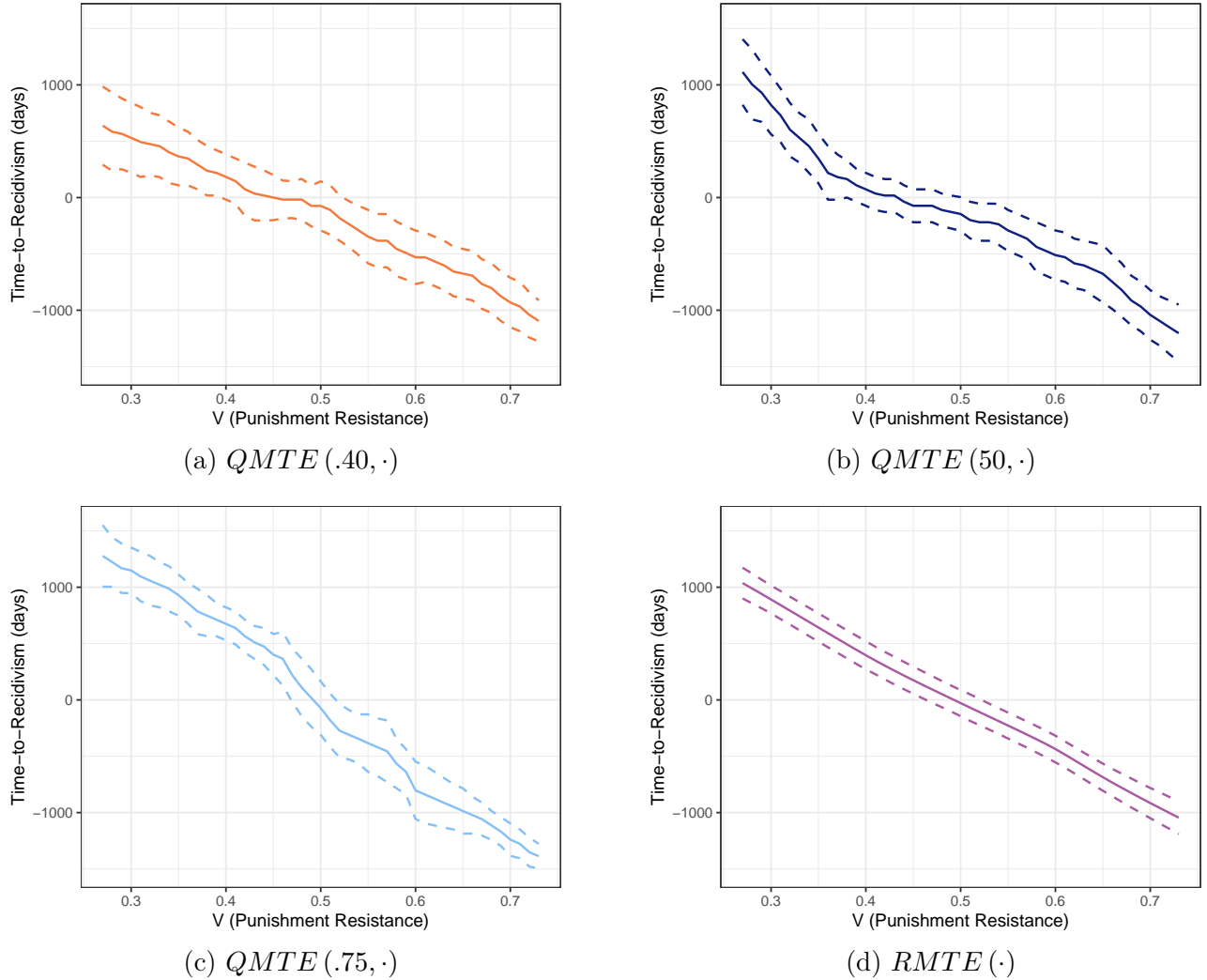
(d) $RMTE(\cdot)$

Notes: Solid lines are the point estimates for the average $QMTE(\tau, \cdot)$ and $RMTE(\cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clusterized at the court district level (Subsection 4.2) using 399 repetitions.

D.2 Robustness Check: Cases after 2014

In this exercise, we verify the robustness of our empirical results to violations of Assumption 5 (Random Censoring). The main concern is that the potential recidivism status is not stationary, i.e., its distribution varies over time. If this is the case, then the $DMTE$ and $QMTE$ functions may vary when we change the sampling period.

In particular, temporal changes in the distribution of potential recidivism may be due to changes in the public defender office. For example, better access to publicly provided high-quality defense may boost the defendant's fairness perception and incentivize them to behave properly after the trial. If this is the case, then the potential recidivism distribution will be different in scenarios with a small or a large number of public defenders. This setting is particularly relevant in our empirical context because the number of public defends varies substantially in the state of Sao Paulo during our sample period: 500 in 2011, 501 in 2012, 610 in 2013, 719 in 2014, 718 in 2015, 717 in 2016, 721 in 2017, 749 in 2018 and 748 in 2019.

Having in mind the discrete jump in the number of public defenders in 2014, we re-estimate our target parameter using a sample that starts in 2014. Naturally, we cannot re-estimate all the $DMTE$ and $QMTE$ functions that we estimated in Sections 5.2.2.1 and 5.2.2.2 because we have a shorter sample period. For this reason, we re-estimate the $DMTE$ function for the first four years and the $QMTE$ function for the first decile, the first quartile and the median.

Figure D.6 presents these results. They are very similar to the results in Sections 5.2.2.1 and 5.2.2.2, indicating the robustness of our results against violations of Assumption 5 (Random Censoring).

Figure D.6: $DMTE$ and $QMTE$ results using cases that started after 2014

(a) $DMTE\,(y,\cdot)$ for $y \in \{1,2,3,4\}$ (b) $QMTE\,(\tau,\cdot)$ for $\tau \in \{.10,.25,.50\}$

Notes: In Figure D.6a, solid lines are the point estimates for the average $DMTE\,(y,\cdot)$ functions indicated in the legend. In Figur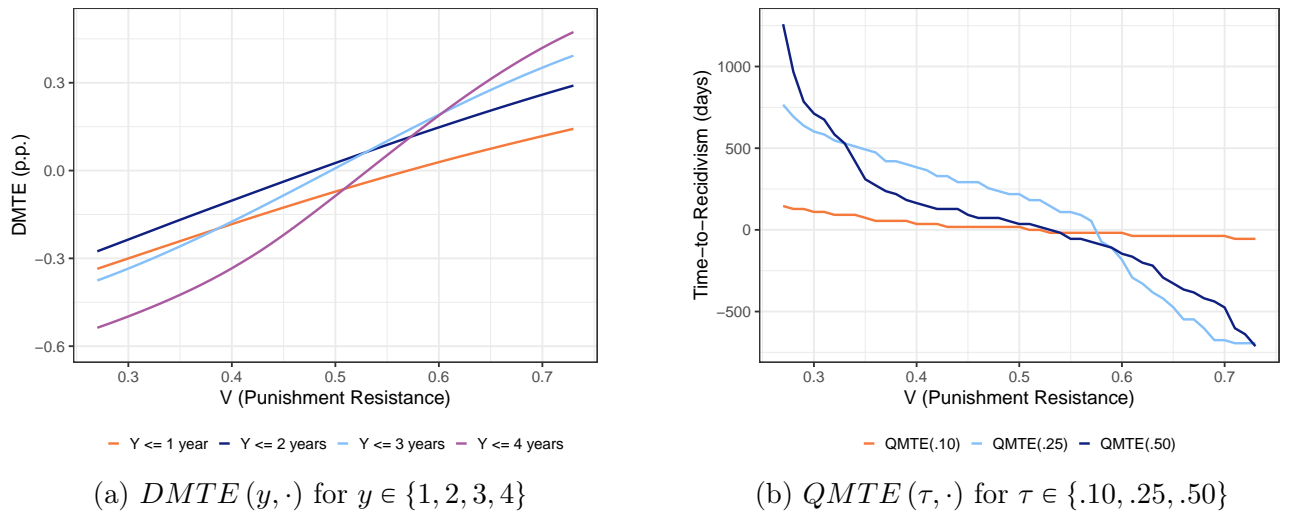e D.6b, solid lines are the point estimates for the average $QMTE\,(\tau,\cdot)$ functions indicated in the legend. All results are based on Corollary 4.1.

E   Relevance of MTE for Duration Outcomes

In this appendix, we justify focusing on the marginal treatment effect (MTE) for duration outcomes using two arguments.

In Appendix E.1, we develop a theoretical model with a policymaker who selects a treatment assignment rule that minimizes the cost of recidivism for the target population of defendants.

In Appendix E.2, we provide a simple example where the treatment benefits most agents in our population. In this example, our proposed focus on quantile treatment effects for duration outcomes correctly highlights that this treatment benefits society. However, focusing on short-time horizons, as usually done in the crime economics literature, leads to the opposite conclusion.

E.1   Theoretical justification of relevance of MTE for duration Outcomes

Following Kitagawa and Tetenov (2018), the policymaker has to choose a treatment rule that determines whether individuals with variables $W = \{Z, V, C\}$ in our target population will be assigned to the treatment group or the comparison group. The policymaker chooses non-randomized treatment rules described by decision sets $G \subset \mathcal{W}$, where $\mathcal{W}$ is the support of W. These decision sets determine the group of individuals $\{W \in G\}$ to whom treatment is assigned. We denote the collection of candidate treatment rules by $\mathcal{G} = \{G \subset \mathcal{W}\}$.

The policymaker's goal in our context is to select a treatment assignment rule that minimizes the cost of recidivism for the target population of defendants. Assuming that the policymaker discounts cost inter-temporally, she chooses the treatment rule that maximizes Y^* for each individual in the target population.

Specifically, we impose that the policymaker chooses the decision set $G \in \mathcal{G}$ that minimizes

$$K(G) := \mathbb{E}\left[\ln\left\{b^{[Y^*(1)\cdot\mathbf{1}\{W\in G\}+Y^*(0)\cdot\mathbf{1}\{W\notin G\}]} \cdot k\right\}\right]$$

where $k \in \mathbb{R}_+$ is the fixed cost of recidivism and $b \in (0,1)$ is the policymaker's discount rate. Rearranging the last equation, we find that

$$K(G) = \ln\{b\} \cdot \mathbb{E}\left[Y^*(1) \cdot \mathbf{1}\{W \in G\} + Y^*(0) \cdot \mathbf{1}\{W \notin G\}\right] + \ln\{k\}$$

$$= \ln\{b\} \cdot \mathbb{E}\left[(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}\right] + \ln\{b\} \cdot \mathbb{E}\left[Y^*(0)\right] + \ln\{k\}$$

Consequently, the policymaker's problem is equivalent to

$$\max_{G \in \mathcal{G}} \mathbb{E}\left[(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}\right].$$

Moreover, note that

$$\mathbb{E}\left[(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}\right]$$

$$= \mathbb{E}\left[\mathbb{E}\left[\left(Y^{*}\left(1\right) - Y^{*}\left(0\right)\right) \cdot \mathbf{1}\left\{W \in G\right\}\vert V, Z, C\right]\right]$$

by the Law of Iterated Expectations

$$= \mathbb{E}\left[\mathbb{E}\left[\left(Y^{*}\left(1\right) - Y^{*}\left(0\right)\right)\vert V, Z, C\right] \cdot \mathbf{1}\left\{W \in G\right\}\right]$$

$$= \mathbb{E}\left[\mathbb{E}\left[\left(Y^{*}\left(1\right) - Y^{*}\left(0\right)\right)\vert V, C\right] \cdot \mathbf{1}\left\{W \in G\right\}\right]$$

by Assumption 1

$$= \mathbb{E}\left[\mathbb{E}\left[\left(Y^{*}\left(1\right) - Y^{*}\left(0\right)\right)\vert V\right] \cdot \mathbf{1}\left\{W \in G\right\}\right]$$

by Assumption 5

$$= \mathbb{E}\left[MTE\left(V\right) \cdot \mathbf{1}\left\{W \in G\right\}\right].$$

Therefore, the policymaker's problem is equivalent to

$$\max_{G \in \mathcal{G}} \mathbb{E}\left[MTE\left(V\right) \cdot \mathbf{1}\left\{W \in G\right\}\right],$$

implying that focusing on the MTE of duration outcomes is relevant when the policymaker wishes to minimize the cost of recidivism over time. Importantly, we discuss how to identify the MTE of duration outcomes in Appendix H.

E.2 Illustrating the relevance of duration outcomes

When analyzing the impact of judicial decisions on recidivism, many authors (Agan et al., 2023; Bhuller et al., 2020; Giles, 2023; Huttunen et al., 2020; Klaassen, 2021; Possebom, 2023) focus on a short time horizon, using a small set of outcome variables that indicate whether the defendant recidivated within a pre-specified number of years. In this paper, we advocate moving beyond this short time horizon and focusing on quantile or average treatment effects of duration outcomes.

In this appendix, we illustrate why focusing on duration outcomes may provide more information than the standard approach in the empirical literature in crime economics. To do so, we abstract from the MTE heterogeneity (variable V) and focus exclusively on the heterogeneity arising from the distribution of the potential outcomes $(Y^{*}\left(0\right), Y^{*}\left(1\right))$.

We illustrate the relevance of quantile and average treatment effects of duration outcomes by analyzing a simple example with discrete random variables. In this example, focusing on short-term outcomes or long-term quantile treatment effects lead to different conclusions about our policy of interest.

We denote potential time-to-recidivism by $Y^{*}\left(0\right)$ and $Y^{*}\left(1\right)$ and measure it in years. Table E.1 shows the joint probability mass function of $(Y^{*}\left(0\right), Y^{*}\left(1\right))$ and their marginal distributions.

Note that, in this example, our judicial decision benefits most defendants. For instance, this treatment strictly increases time-to-recidivism for 50% of the defendants $(Y^{*}\left(0\right) < Y^{*}\left(1\right))$.

Table E.1: Joint Probability of $(Y^*(0), Y^*(1))$ and their Marginal Distributions

	$\mathbb{P}[Y^*(0) = \cdot, Y^*(1) = \cdot]$	$Y^*(0) =$					$\mathbb{P}[Y^*(1) = \cdot]$
		1	2	3	10	20	
	1	.10	0	.10	0	0	.20
	2	0	.10	.10	0	0	.20
$Y^*(1) =$	3	0	0	0	0	0	0
	10	0	0	0	.10	0	.10
	20	.05	.05	0	.40	0	.50
	$\mathbb{P}[Y^*(0) = \cdot]$	.15	.15	.20	.50	0	1

Note: The last column reports the marginal distribution of $Y^*(1)$. The last row reports the marginal distribution of $Y^*(0)$. The cells in the center of the table report the joint distribution of $(Y^*(0), Y^*(1))$.

Moreover, only 20% of the defendants are harmed by this treatment $(Y^*(0) > Y^*(1))$.

However, a short-time horizon analysis would conclude that this treatment is harmful. For example, this treatment increases the probability of recidivism within one year by 5 p.p. and the probability of recidivism within two years by 10 p.p, i.e., $\mathbb{P}[Y^*(1) \leqslant 1] - \mathbb{P}[Y^*(0) \leqslant 1] = 0.05$ and $\mathbb{P}[Y^*(1) \leqslant 2] - \mathbb{P}[Y^*(0) \leqslant 2] = 0.1$.

Differently from the standard empirical analysis, we advocate for focusing on quantile and average treatment effects of duration outcomes. For example, the Quantile Treatment Effect on the Median is equal to seven years because the median of $Y^*(1)$ equal ten years and the median of $Y^*(0)$ equals three years. Moreover, the average treatment effect equals 5.55 years in this example.

Therefore, our proposed analysis would correctly highlight that this treatment benefits at least some agents in our society.

F Identification without Restrictions on Censoring

In this appendix, we focus on which parameters can be point-identified when we do not impose any restriction on the relationship between the censoring variable and the potential outcomes. To compensate for not imposing Assumption 5 nor Assumptions G.1-G.2, we need to allow the $DMTR$ function to depend on the censoring variable.

Specifically, our causal parameter is given by:

$$DMTR_d\left(y,v,c\right) \coloneqq \mathbb{P}\left[Y^*(d) \leqslant y \middle| V = v, C = c\right]$$

for any $d \in \{0,1\}$, $y < \gamma_C$, $v \in [0,1]$ and $c \in \mathcal{C}$. Note that our causal parameter is interpretable as a conditional distributional marginal treatment response. In particular, the censoring variable C acts similarly to a covariate in the standard MTE analysis (Carneiro et al., 2011).

In our empirical application, conditioning on the censoring variable is equivalent to conditioning on the defendant cohort or time-fixed effects. Considering that most studies about judicial decisions (Agan et al., 2023; Bhuller et al., 2020; Huttunen et al., 2020; Klaassen, 2021) condition on district-by-time fixed effects, they identify the conditional $DMTR$ function for a pre-specified value of y. In this appendix, we discuss how to extend their analysis to consider conditional quantile marginal treatment effects and marginal treatment effects (Remark 4). We also discuss the advantages of formally addressing censoring by imposing Assumption 5 (Remark 5). Lastly, we discuss the implicit assumptions behind this standard approach in the empirical literature (Remark 6).

To point-identify the conditional $DMTR$ function, we eliminate Assumption 5 and impose Assumptions 1-4 only.

Proposition F.1. *If Assumptions 1-4 hold, then*

$$DMTR_d\left(y,p,y+\delta\right) = (2d-1)\cdot\frac{\partial\mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z,C\right) = p, C = y+\delta\right]}{\partial v}$$

for any $d \in \{0,1\}$, $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

Remark 4. A direct consequence of Proposition F.1 is the identification of the quantile marginal treatment response function $QMTR_d\left(\tau,p,y+\delta\right)$ conditional on the censoring variable for any $\tau \in (0,\overline{\tau}_d\left(p,y+\delta\right))$, where $\overline{\tau}_d\left(p,y+\delta\right) \coloneqq DMTR_d\left(\gamma_C,p,y+\delta\right)$. Additionally, if we impose Assumptions H.1 and H.2, then we straight-forwardly identify the MTE function conditional on the censoring variable.

Remark 5. The comparison between Propositions 3.1 and F.1 illustrates the identifying power of Assumption 5. It allows us to combine multiple values of the censoring variable to identify a single point of the $DMTR$ function through the integral of $\frac{\partial\mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z,C\right) = p, C = y+\delta\right]}{\partial v}$ over different values of δ. In our empirical application, it means that we can combine multiple defendant cohorts to identify a single evaluation

point in the $DMTR$ function, increasing statistical power by being transparent about censoring assumptions.

In contrast, the standard empirical practice frequently defines a set of time periods, creates binary outcome variables for each time horizon, and only keeps observations that are observed for a period longer than their longest time horizon of interest. Due to this desire to keep the sample consistent across outcome variables, the existing empirical practice effectively reduces power when analyzing shorter horizons.

Remark 6. Proposition F.1 controls for time effects non-linearly without relying on implicit separability assumptions. In contrast, existing empirical strategies focusing on time to event-date make use of linear models with time-fixed effects. Hence, they rely on implicit separability assumptions. In case of failure of such assumptions, the estimands from these time-fixed effects models are unlikely to be a relevant causal parameter.

Differently from this standard empirical practice, our approach recovers causal effects of interest by bypassing separability assumptions and relying on Random Censoring. We do so by connecting time effects and treatment effects in an explicit and transparent way under the assumption that there are no endogenous time effects once we control for the individual latent punishment resistance.

Proof. For brevity, we show the proof of Proposition F.1 when $d = 1$.

Fix $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$. Note that

$$\mathbb{P}\left[Y \leqslant y, D = 1 \middle| P\left(Z, C\right) = v, C = y + \delta\right]$$

$$= \mathbb{E}\left[\mathbf{1}\left\{Y \leqslant y\right\}\mathbf{1}\left\{P\left(Z, C\right) \geqslant V\right\}\middle| P\left(Z, C\right) = v, C = y + \delta\right]$$

by (3.1)

$$= \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\}\mathbf{1}\left\{p \geqslant V\right\}\middle| P\left(Z, y + \delta\right) = p, C = y + \delta\right]$$

because Y_1^* is not censored when $C > y$

$$= \int_0^1 \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\}\mathbf{1}\left\{p \geqslant v\right\}\middle| P\left(Z, y + \delta\right) = p, C = y + \delta, V = v\right]dv$$

by the Law of Iterated Expectations and Assumption 3

$$= \int_0^1 \mathbf{1}\left\{p \geqslant v\right\}\mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\}\middle| P\left(Z, y + \delta\right) = p, C = y + \delta, V = v\right]dv$$

$$= \int_0^p \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\}\middle| P\left(Z, y + \delta\right) = p, C = y + \delta, V = v\right]dv$$

$$= \int_0^p \mathbb{P}\left[Y^*(1) \leqslant y\middle| C = y + \delta, V = v\right]dv$$

by Assumption 1.

Consequently, the Leibniz Integral Rule implies that

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 1 \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial v} = \mathbb{P}\left[Y^*(1) \leqslant y \middle| C = y + \delta, V = p\right]$$

$$= DMTR_1\left(y, p, y + \delta\right).$$

We can prove the same result for $d = 0$ analogously. ∎

G Partial Identification Strategies

In some empirical applications, Assumption 5 may be too strong, while in others, it may be plausible. Since this is very context-specific, it is worth coming up with alternative identification strategies that accommodate dependent censoring mechanisms. In this appendix, we discuss two alternative assumptions that restrict the dependence between the censoring variable and the latent heterogeneity, but not to the point of imposing censoring independence. Section G.1 imposes that potential outcomes are negatively regression-dependent on the censoring variable, while Section G.2 intuitively imposes that the censoring problem is not too severe.

G.1 Partial identification under regression dependence

In this subsection, we impose that potential outcomes are negatively regression-dependent on the censoring variable. This alternative assumption restricts the relationship between the latent heterogeneity, the censoring variable, and the potential outcomes.[27]

Assumption G.1 (Regression Dependence). *Conditional on V, the potential outcomes are negatively regression dependent on the censoring variable, i.e., $\mathbb{P}\left[Y^*(d) \leqslant y \mid C = \tilde{c}, V = v\right] \geqslant \mathbb{P}\left[Y^*(d) \leqslant y \mid C = c, V = v\right]$ for any $d \in \{0,1\}$, any $v \in (0,1)$ and any $(c,\tilde{c}) \in \mathcal{C}^2$ such that $c \leqslant \tilde{c}$.*

In our empirical application, Assumption G.1 imposes that the potential outcomes of more recent cases first-order stochastically dominate the potential outcomes of older cases. Intuitively, this restriction imposes that defendants are committing fewer crimes over time and is plausible given that the state of São Paulo became safer during our sampling period.[28]

To derive bounds around the $DMTR$ functions, define the following auxiliary quantities:

$$
\begin{aligned}
LB_d(y, v, \delta) &:= \mathbb{P}(y + \delta \leqslant C) \cdot (2d - 1) \cdot \gamma_d(y, v, y + \delta) \\
UB_d(y, v, \delta) &:= \mathbb{P}(C \leqslant y) + \mathbb{P}(y + \delta \leqslant C) \\
&\quad + \mathbb{P}(y \leqslant C \leqslant y + \delta) \cdot (2d - 1) \cdot \gamma_d(y, v, y + \delta),
\end{aligned}
$$

where $\delta \in \mathbb{R}_{++}$. The next proposition describes the bounds around the $DMTR$ functions when potential outcomes are negatively regression-dependent on the censoring variable.

Proposition G.1. *Suppose that Assumptions 1-4 and G.1 hold. Then,*

$$
DMTR_d(y, v) \in \left[\max_{\delta \in \mathcal{D}} LB_d(y, v, \delta), \min_{\delta \in \mathcal{D}} UB_d(y, v, \delta)\right]
$$

---

[27]Related assumptions have been used by Chesher (2005), Jun, Pinkse and Xu (2011), and Kedagni and Mourifie (2014) in different contexts. For more information on the definition of regression dependence and other concepts of statistical dependence, see Lehmann (1966).

[28]In a different empirical context, positive regression dependence may be more plausible than negative regression dependence. Similar bounds can be derived based on this alternative assumption.

for any $d \in \{0,1\}$, $y < \gamma_C$ and $v \in \mathcal{P}$, where $\mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$.

Proof. See Appendix G.1.1. ∎

First, one can see that the bounds in Proposition G.1 do not "collapse" to point identification when Assumption 5 holds. That is because the regression-dependence in Assumption G.1 is compatible with dependent censoring but does not directly restrict the amount of dependence between the potential outcomes and the censoring random variable. In order words, the nature of Assumption G.1 is different from Assumption 5 and does not constitute "continuous relaxations" of the independence assumption.

Second, Assumption G.1 allows us to exploit the information in $\gamma_d(y, v, y + \delta)$ even under dependent censoring because of the (stochastic) monotonicity in C. Since this monotonicity property holds for different values of C less than the $\tilde{c}$, we take the supremum and infimum over δ, so the bounds are tighter.

Third, the identification region will functionally depend on the propensity score (and thus the instrument), as reflected in the presence of $\gamma_d(y, v, y+\delta)$ in the bounds. This will determine the shape of it.

Furthermore, the bounds' length depends on the proportion of censored observations close to the value of the particular y, reflecting that regions with heavier censoring data tend to have wider bounds.

From the partial identification of the $DMTR_d(y, v)$ functions, it also follows the partial identification of a range of $QMTE(\tau, v)$ across τ and the $RMTE(v)$, just like before. If one further imposes Assumptions H.1 and H.2, partial identification results for the MTE function will also follow. For these functions, though, it is important to ensure that the lower and upper bounds in Proposition G.1 are monotone in y, which can be enforced using a similar approach as in Manski and Molinari (2021). More specifically, for a grid of weakly increasing y's, if $\max_{\delta \in \mathcal{D}} LB_d(y_{k+1}, c, \delta) < \max_{\delta \in \mathcal{D}} LB_d(y_k, c, \delta)$, we can simply redefine $\max_{\delta \in \mathcal{D}} LB_d(y_{k+1}, c, \delta) = \max_{\delta \in \mathcal{D}} LB_d(y_k, c, \delta)$; the analogous is true for the upper bound. Alternatively, one can use the rearrangement procedure as in Chernozhukov et al. (2009). We state these results as corollaries for convenience.

Corollary G.1. *Suppose that Assumptions 1-4, B.7 and G.1 hold. Then,*

(a) $QMTE(\tau, v)$ is partially identified for any $v \in \mathcal{P}$ and $\tau \in (0, \overline{\tau}(v))$.

(b) the $RMTE(v)$ function is partially-identified for any $v \in \mathcal{P}$.

Corollary G.2. *If Assumptions Assumptions 1-4, G.1, H.1 and H.2 hold, then $MTE(v)$ is partially identified for any $v \in \mathcal{P}$.*

Finally, the bounds in Proposition G.1 can be estimated using methods similar to the methods described in Subsection 4.2. The main difference between the estimators of the bounds and

the point-estimators (Subsection 4.2) is that, when estimating the bounds, we take either the maximum or the minimum over values of c in Step 5 instead of taking the mean. Consequently, these estimators will converge in probability to the bounds in Proposition G.1.

G.1.1 Proof of Proposition G.1

Fix $d \in \{0, 1\}$, $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

Note that Equations (A.1) and (A.2) imply that

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 1 \mid P\left(Z, C\right) = v, C = y + \delta\right]}{\partial v} = \mathbb{P}\left[Y^*(1) \leqslant y \mid C = y + \delta, V = v\right] \tag{G.1}$$

and

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 0 \mid P\left(Z, C\right) = v, C = y + \delta\right]}{\partial v} = -\mathbb{P}\left[Y^*(0) \leqslant y \mid C = y + \delta, V = v\right] \tag{G.2}$$

according to the Leibniz Integral Rule.

Combining the last two equations, we have that

$$\mathbb{P}\left[Y^*(d) \leqslant y \mid C = y + \delta, V = v\right] = (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \mid P\left(Z, C\right) = v, C = y + \delta\right]}{\partial v}. \tag{G.3}$$

Moreover, observe that:

$$\mathbb{P}\left[Y^*(d) \leqslant y \mid V = v\right]$$

$$= \int \mathbb{P}\left[Y^*(d) \leqslant y \mid C = \tilde{c}, V = v\right] f_{C|V}\left(\tilde{c}|v\right) d\tilde{c}$$

by the Law of Iterated Expectations

$$= \int \mathbb{P}\left[Y^*(d) \leqslant y \mid C = \tilde{c}, V = p\right] f_C\left(\tilde{c}\right) d\tilde{c}$$

because $V \perp\!\!\!\perp C$ by Assumption 3

$$= \int_0^y \mathbb{P}\left[Y^*(d) \leqslant y \mid C = \tilde{c}, V = p\right] f_C\left(\tilde{c}\right) d\tilde{c}$$

$$+ \int_y^{y+\delta} \mathbb{P}\left[Y^*(d) \leqslant y \mid C = \tilde{c}, V = p\right] f_C\left(\tilde{c}\right) d\tilde{c}$$

$$+ \int_{y+\delta}^{+\infty} \mathbb{P}\left[Y^*(d) \leqslant y \mid C = \tilde{c}, V = p\right] f_C\left(\tilde{c}\right) d\tilde{c},$$

implying, by Assumption G.1, that

$$\mathbb{P}\left[Y^*(d) \leqslant y \mid V = p\right] \leqslant \mathbb{P}(C \leqslant y) + \mathbb{P}(y + \delta \leqslant C)$$

$$+ \mathbb{P}(y \leqslant C \leqslant y + \delta)\mathbb{P}\left[Y^*(d) \leqslant y \mid C = y + \delta, V = p\right] \tag{G.4}$$

and

$$\mathbb{P}\left[Y^*(d) \leqslant y \mid V = p\right] \geqslant \mathbb{P}(y + \delta \leqslant C)\mathbb{P}\left[Y^*(d) \leqslant y \mid C = y + \delta, V = p\right] \tag{G.5}$$

Thus, combining Equations (G.4) and (G.5) with (G.3), we have that

$$DMTR_d(y, v)$$
$$\in \left[\mathbb{P}(y + \delta \leqslant C) \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \mid P(Z, C) = v, C = y + \delta\right]}{\partial v}, \right.$$
$$\mathbb{P}(C \leqslant y) + \mathbb{P}(y + \delta \leqslant C)$$
$$\left. + \mathbb{P}(y \leqslant C \leqslant y + \delta) \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \mid P(Z, C) = v, C = y + \delta\right]}{\partial v} \right].$$

Since the bounds above hold for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$DMTR_d(y, v)$$
$$\in \left[\max_{\delta \in \mathcal{D}} \left\{ \mathbb{P}(y + \delta \leqslant C) \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \mid P(Z, C) = v, C = y + \delta\right]}{\partial v} \right\}, \right.$$
$$\left. \min_{\delta \in \mathcal{D}} \left\{ \begin{array}{c} \mathbb{P}(C \leqslant y) + \mathbb{P}(y + \delta \leqslant C) + \mathbb{P}(y \leqslant C \leqslant y + \delta) \\ \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \mid P(Z, C) = v, C = y + \delta\right]}{\partial v} \end{array} \right\} \right],$$

where $\mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$.

G.2 Partial identification under a continuous violation of random censoring

In this subsection, we impose that the conditional distribution of the potential outcomes given the censoring variable and the latent heterogeneity variable is close to the conditional distribution of the potential outcomes given only the latent heterogeneity variable.[29] Differently from Assumption G.1, the following assumption constitutes a "continuous relaxation" of censoring independence (Assumption 5).

Assumption G.2 ("Continuous Relaxation"). *The conditional distribution of the potential outcomes given the censoring variable and the latent heterogeneity variable is similar to the conditional distribution of the potential outcomes given only the latent heterogeneity variable i.e., there exists $\overline{B} \in \mathbb{R}_{++}$ such that*

$$\left| \mathbb{P}\left[Y^*(d) \leqslant y \mid C = y + \delta, V = v\right] - \mathbb{P}\left[Y^*(d) \leqslant y \mid V = v\right] \right| \leqslant \overline{B}$$

for any $y \in \mathcal{Y}$, $v \in [0, 1]$, $\delta \in \mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$ and $d \in \{0, 1\}$.

Using this assumption, we can derive bounds around the $DMTR$ functions.

Proposition G.2. *Suppose that Assumptions 1-4 and G.2 hold. Then,*
$$DMTR_d(y, v) \in \left[LB_d\left(y, v, \overline{B}\right), UB_d\left(y, v, \overline{B}\right) \right]$$

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[29]A similar assumption is used by Kline and Santos (2013) in a sample selection context.

where

$$LB_d\left(y, v, \overline{B}\right) := -\overline{B} + \left(2\dot{d} - 1\right) \cdot \max_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta)$$

and

$$UB_d\left(y, v, \overline{B}\right) := \overline{B} + \left(2\dot{d} - 1\right) \cdot \min_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta)$$

for any $d \in \{0, 1\}$, $y < \gamma_C$ *and* $v \in \mathcal{P}$.

Proof. See Appendix G.2.1. ∎

First, the bounds in Proposition G.2 can be estimated using methods similar to the methods described in Subsection 4.2. The main difference between the estimators of the bounds and the point-estimators (Subsection 4.2) is that, when estimating the bounds, we take either the maximum or the minimum over values of c in Step 5 instead of taking the mean. Consequently, these estimators will converge in probability to the bounds in Proposition G.2.

Second, differently from Proposition G.1, the bounds in Proposition G.2 "collapse" to point identification when Assumption 5 holds. In this case, we have that $\overline{B} = 0$ and $\max_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta) = \min_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta)$.

Third, we take the supremum and infimum over δ to tighten the bounds because our continuous relaxation of censoring independence holds for every value of C greater than y.

Finally, the identification region will functionally depend on the propensity score (and thus the instrument), as reflected in the presence of $\gamma_d(y, v, y + \delta)$ in the bounds. This will determine the shape of it.

Furthermore, the bounds' length depends on the choice of $\overline{B}$. We recommend choosing $\overline{B}$ based on the bounds around the $DMTE$ function and defined in Corollary G.3.

Corollary G.3. *Suppose that Assumptions 1-4 and G.2 hold. Then,*
$$DMTE\left(y, v\right) \in \left[\underline{\Delta}\left(y, v, \overline{B}\right), \overline{\Delta}\left(y, v, \overline{B}\right)\right]$$
where
$$\underline{\Delta}\left(y, v, \overline{B}\right) := LB_1\left(y, v, \overline{B}\right) - UB_0\left(y, v, \overline{B}\right)$$
and
$$\overline{\Delta}\left(y, v, \overline{B}\right) := UB_1\left(y, v, \overline{B}\right) - LB_0\left(y, v, \overline{B}\right)$$
*for any* $y < \gamma_C$ *and* $v \in \mathcal{P}$.

Corollary G.3 may be used to choose $\overline{B}$ according to a breakdown analysis (Kline and Santos, 2013; Masten and Poirier, 2018). For example, the researcher may be particularly interested in $DMTE\left(\overline{y}, \cdot\right)$ for some value of $\overline{y} \in \mathcal{Y}$. If, based on Proposition 3.1, $DMTE\left(\overline{y}, v\right) \neq 0$ for some $v \in \mathcal{P}$, then the researcher can choose the smallest value of $\overline{B}\left(\overline{y}\right)$ such that the bounds in Corollary G.3 contain the zero function, i.e., $0 \in \left[\underline{\Delta}\left(\overline{y}, v, \overline{B}\left(\overline{y}\right)\right), \overline{\Delta}\left(\overline{y}, v, \overline{B}\left(\overline{y}\right)\right)\right]$ for every

$v \in \mathcal{P}$. This value of $\overline{B}(\overline{y})$ is known as the breakdown point.[30]

Figure G.1 shows the Breakdown Points $\overline{B}(y)$ for $y < 8$ years. Note that the breakdown points for short-run recidivism (small values of y) are smaller than the breakdown point for mid-run recidivism (values of y around 2000 days or 5 years approximately). This finding suggests that our results for mid-run recidivism are more robust to violations of the random censoring assumption than our results for short-run recidivism.

Figure G.1: Breakdown Point $\overline{B}(y)$ for $y < \gamma_C$
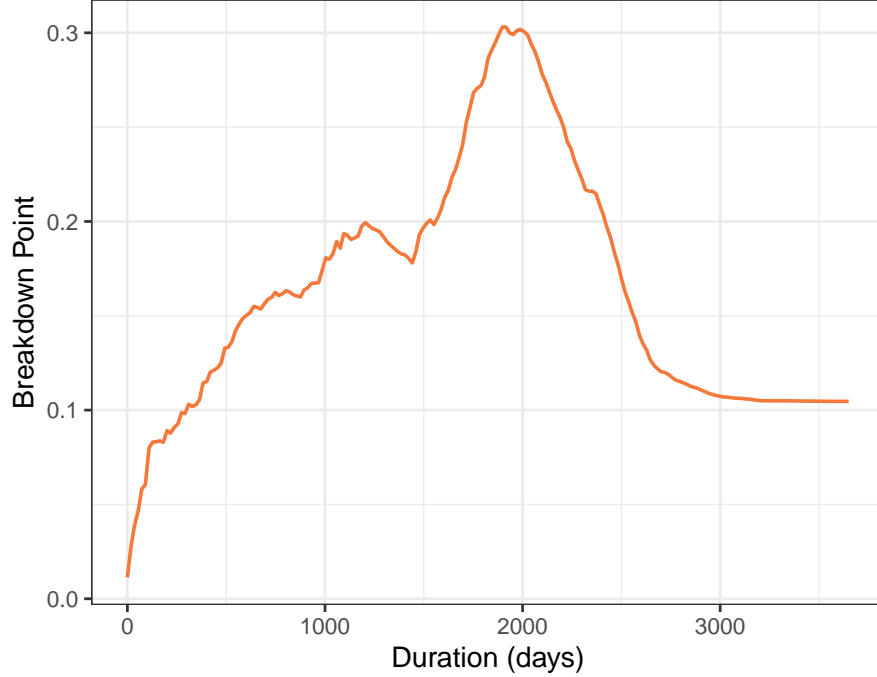
Notes: For $y < 8$ years, the breakdown point $\overline{B}(y)$ is the smallest value of $\overline{B}(y)$ such that the bounds in Corollary G.3 contain the zero function, i.e., $0 \in \left[\underline{\Delta}(y, v, \overline{B}(y)), \overline{\Delta}(y, v, \overline{B}(y))\right]$ for every $v \in \mathcal{P}$.
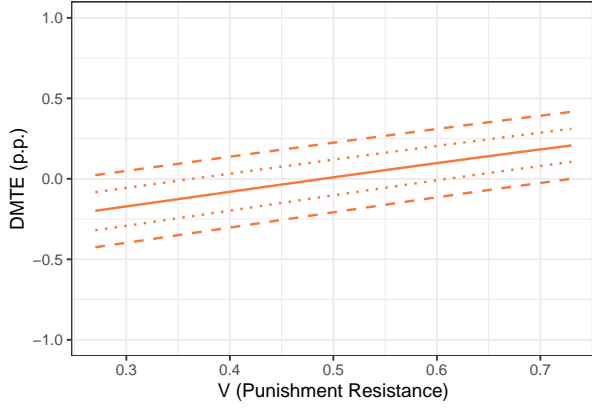
More importantly, Figure G.1 suggests that the results in Section 5.2.2.1 are robust to violations of the random censoring assumption. Considering that the largest possible value of the breakdown point is one, we believe that a breakdown point larger than 0.1 (i.e., $\overline{B}(y) \geqslant 0.1$) suggests that the estimated $DMTE(y, \cdot)$ is robust against continuous relaxations of Assumption 5. We highlight that this condition holds for most time horizons in Figure G.1.

To directly inspect the bounds in Corollary G.3, we can plot them for a few values of $\overline{B}$. Figures G.2 and G.3 plot $\underline{\Delta}(y, \cdot, \overline{B})$ and $\overline{\Delta}(y, \cdot, \overline{B})$ for $\overline{B} \in \left\{\overline{B}(y)/2, \overline{B}(y)\right\}$. Solid lines are the point estimates for the average $DMTE(y, \cdot)$ functions indicated in the caption of each subfigure (Corollary 4.1). Dashed lines are the bounds around the average $DMTE(y, \cdot)$ functions that use the estimated breakdown point in their construction, i.e., $\underline{\Delta}\left(y, \cdot, \overline{B}(y)\right)$ and $\overline{\Delta}\left(y, \cdot, \overline{B}(y)\right)$. Dotted lines are the bounds that use half of the estimated breakdown point in their construction,

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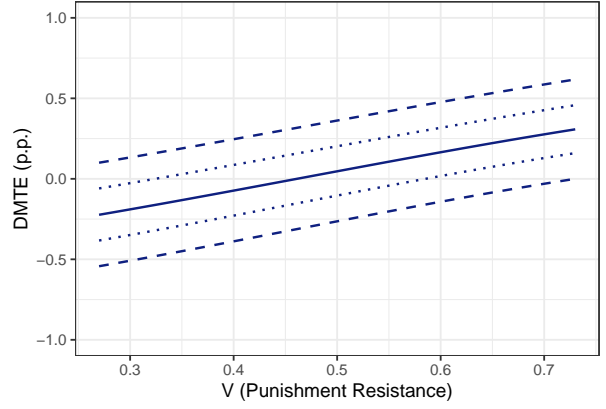[30]Conducting inference about the breakdown point is beyond the scope of this paper.

i.e., $\underline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)/2\right)$ and $\overline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)/2\right)$.

Figure G.2: Bounds around $DMTE\left(y, \cdot\right)$ for $y \in \{1, 2, 3, 4\}$

(a) $DMTE\left(1, \cdot\right)$

(b) $DMTE\left(2, \cdot\right)$

(c) $DMTE\left(3, \cdot\right)$

(d) $DMTE\left(4, \cdot\right)$

Notes: Solid lines are the point estimates for the average $DMTE\left(y, \cdot\right)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Dashed lines are the bounds around the average $DMTE\left(y, \cdot\right)$ functions that use the estimated breakdown point in their construction, i.e., $\underline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)\right)$ and $\overline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)\right)$. Dotted lines are the bounds around the average $DMTE\left(y, \cdot\right)$ functions that use half of the estimated breakdown point in their construction, i.e., $\underline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)/2\right)$ and $\overline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)/2\right)$. All bounds are based on Corollary G.3.

When focusing on $\underline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)/2\right)$ and $\overline{\Delta}\left(y, \cdot, \overline{B}\left(y\right)/2\right)$ (dotted lines), we find that the results in Section 5.2.2.1 are robust to violations of the random censoring assumption. For example, it is not possible to fit a flat line between these bounds. This finding suggests that there is heterogeneity with respect to the punishment resistance even when we allow for continuous relaxations of Assumption 5.

Beyond discussing bounds around the $DMTE$ functions and their breakdown points, the partial identification of the $DMTR_d\left(y, v\right)$ functions (Proposition G.2) also implies the partial identification of a range of $QMTE\left(\tau, v\right)$ across τ and the $RMTE(v)$, just like before. If one

Figure G.3: Bounds around $DMTE(y, \cdot)$ for $y \in \{5, 6, 7, 8\}$

(a) $DMTE(5, \cdot)$

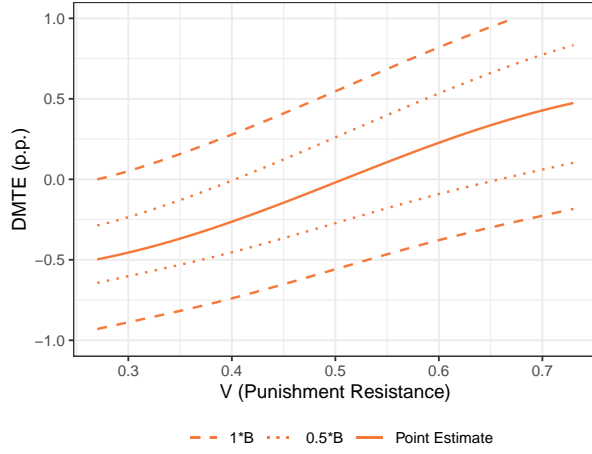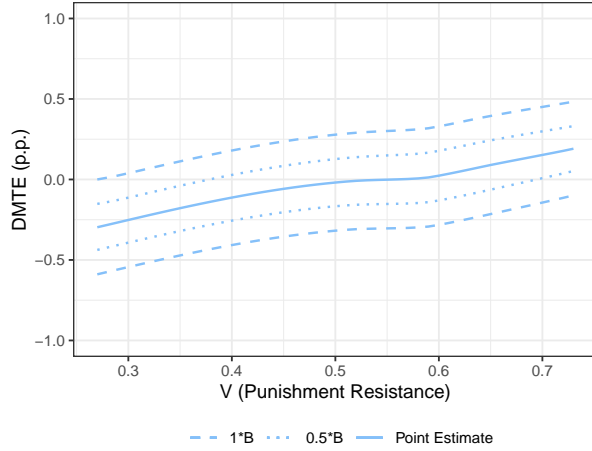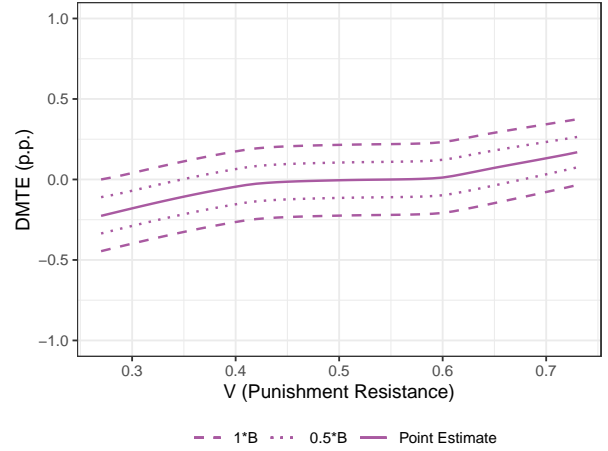(b) $DMTE(6, \cdot)$

(c) $DMTE(7, \cdot)$

(d) $DMTE(8, \cdot)$

Notes: Solid lines are the point estimates for the average $DMTE(y, \cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Dashed lines are the bounds around the average $DMTE(y, \cdot)$ functions that use the estimated breakdown point in their construction, i.e., $\underline{\Delta}(y, \cdot, \overline{B}(y))$ and $\overline{\Delta}(y, \cdot, \overline{B}(y))$. Dotted lines are the bounds around the average $DMTE(y, \cdot)$ functions that use half of the estimated breakdown point in their construction, i.e., $\underline{\Delta}(y, \cdot, \overline{B}(y)/2)$ and $\overline{\Delta}(y, \cdot, \overline{B}(y)/2)$. All bounds are based on Corollary G.3.

further imposes Assumptions H.1 and H.2, partial identification results for the MTE function will also follow. For these functions, though, it is important to ensure that the lower and upper bounds in Proposition G.2 are monotone in y, which can be enforced using a similar approach as in Manski and Molinari (2021). Alternatively, one can use the rearrangement procedure as in Chernozhukov et al. (2009). We state these results as corollaries for convenience.

Corollary G.4. *Suppose that Assumptions 1-4, B.7 and G.2 hold. Then,*

(a) $QMTE(\tau, v)$ is partially identified for any $v \in \mathcal{P}$ and $\tau \in (0, \overline{\tau}(v))$.

(b) the $RMTE(v)$ function is partially-identified for any $v \in \mathcal{P}$.

Corollary G.5. *If Assumptions Assumptions 1-4, G.2, H.1 and H.2 hold, then $MTE(v)$ is partially identified for any $v \in \mathcal{P}$.*

G.2.1 Proof of Proposition G.2

Fix $d \in \{0, 1\}$, $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

To derive the upper bound, observe that

$$DMTR_d(y, v) = \mathbb{P}\left[Y^*(d) \leqslant y \mid V = v\right]$$

by definition

$$\leqslant \overline{B} + \mathbb{P}\left[Y^*(d) \leqslant y \mid C = y + \delta, V = v\right]$$

according to Assumption G.2

$$= \overline{B} + (2d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \mid P(Z, C) = v, C = y + \delta\right]}{\partial v}$$

according to (G.3)

$$= \overline{B} + (2d - 1) \cdot \gamma_d(y, v, y + \delta)$$

by definition.

Since the bounds above hold for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$DMTR_d(y, v) \leqslant \overline{B} + \left(2\dot{d} - 1\right) \cdot \min_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta).$$

We can derive the lower bound analogously.

H Exploring Average Marginal Treatment Effects

In this appendix, we explore two types of average marginal treatment effects. Section H.1 defines truncated and untruncated versions of the marginal treatment effect. Section H.2 states the assumptions that are necessary to identify the untruncated version of the marginal treatment effect function. Section H.3 describes how to identify both types of marginal treatment effect functions, while Section H.4 describes how to estimate them. Lastly, Section H.5 analyzes our empirical application through the lenses of these types of average marginal treatment effect functions.

H.1 Definition

We define the average marginal treatment response function as

$$AMTR_d(v) \quad := \quad \mathbb{E}\left[Y^*(d)|\, V = v\right], \tag{H.1}$$

where $d \in \{0, 1\}$ and $v \in [0, 1]$. These counterfactual parameters provide the average time-to-recidivism under treatment d among defendants with punishment resistance v.

Based on these counterfactual objects, it is straightforward to define the Average Marginal Treatment Effect function:

$$MTE(v) \quad := \quad AMTR_1(v) - AMTR_0(v) = \int_0^1 QMTE(\tau, v) \, d\tau. \tag{H.2}$$

We also note that one can express $MTE(v)$ as a function of the $DMTE(y, v),$[31]

$$MTE(v) = -\int_{\mathbb{R}_+} DMTE(y, v) \, dy.$$

Positive values of the MTE functions indicate that punishment by fines and community services increases the defendant's time-to-recidivism compared to no punishment (so treatment is working as intended).

Since we are dealing with a duration outcome subject to right-censoring, it is important to recognize that recovering $MTE(v)$ may be challenging, as it requires identifying the potential outcomes' entire (conditional) counterfactual distribution. To somehow sidestep this limitation, it is common in the survival analysis literature to focus on a restricted version of the mean.[32] Following this rationale, we introduce a restricted version of the $MTE(v)$ function below, though we recognize that it is potentially less interesting than the $MTE(v)$.

Let γ_C denote the upper-bound of the support of the censoring variable C, that is, $\gamma_C :=$

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[31]This follows from the fact that, for any non-negative random variable I, $\mathbb{E}[I] = \int_{\mathbb{R}_+} (1 - \mathbb{P}[I \leqslant u]) du.$

[32]See, e.g., Karrison (1987), Zucker (1998), Chen and Tsiatis (2001), Zhang and Schaubel (2012), among many others.

$\inf \left\{ c \in \overline{\mathbb{R}} \colon \mathbb{P}\left[C \leqslant c\right] = 1 \right\}$. Define the restricted AMTR function as

$$RAMTR_d\left(v\right) := \mathbb{E}\left[\min\{Y^*(d), \gamma_C\} \mid V = v\right], \tag{H.3}$$

and the restricted average marginal treatment effect as

$$RMTE\left(v\right) \quad := \quad RAMTR_1\left(v\right) - RAMTR_0\left(v\right). \tag{H.4}$$

The RMTE is also connected with the DMTE function:

$$RMTE(v) = -\int_0^{\gamma_C} DMTE\left(y, v\right) dy,$$

which follows from the fact that, for a generic non-negative outcome W, $\mathbb{E}\left[\min\{W, \gamma_C\} \mid V = v\right] = \int_0^{\gamma_C} \left(1 - \mathbb{P}(W \leqslant y \mid V = v)\right) dy$. Of course, if $\gamma_C = \infty$ or if the support of $Y^*(d)$ is contained in the support of C (for the given v), then $RMTE\left(v\right) = MTE\left(v\right)$.

H.2   Assumptions

Although Assumptions 1-5 are sufficient to identify the RMTE parameter, we require two extra assumptions to identify the MTE function. These additional support restrictions (Assumptions H.1 and H.2) guarantee the identification of the entire DMTE and QMTE functions, implying that the MTE is also identified.[33]

Assumption H.1 (Finite Moments). *Conditional on C, the potential outcome variables have finite first moments, i.e., $\mathbb{E}\left[|Y\left(d\right)| \mid V = v, C = c\right] < \infty$ for any $d \in \{0, 1\}$, any $v \in [0, 1]$ and any $c \in \mathcal{C}$.*

Assumption H.1 is a regularity condition that allows us to apply standard integration theorems and ensures that average treatment effects are well-defined.

Assumption H.2 (Support Restriction). *The support of the uncensored potential outcomes is smaller than the support of the censoring variable, i.e., $\gamma_C = +\infty$ or $\gamma_d < \gamma_C$ for any $d \in \{0, 1\}$, where $\gamma_C := \inf \left\{ c \in \overline{\mathbb{R}} \colon \mathbb{P}\left[C \leqslant c\right] = 1 \right\}$ and $\gamma_d := \inf \left\{ y \in \overline{\mathbb{R}} \colon \mathbb{P}\left[Y^*\left(d\right) \leqslant y\right] = 1 \right\}$ for any $d \in \{0, 1\}$.*

Assumption H.2 restricts the support of the potential outcomes of interest to be smaller than the censoring variable's support. In our empirical application, this assumption imposes that all defendants recidivate within ten years, which is the longest observation period in our sample. Formally, this restriction imposes that $\gamma_d < \gamma_C = 10$ years for any $d \in \{0, 1\}$. This rule out the possibility of defendants not recidivating until they die, and it is therefore not very plausible in our specific contest. We still present results using this assumption as they may be appropriate in empirical contexts different from ours.

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[33]The last support assumption may be restrictive in most applications, but we include it for completeness.

H.3   Identification

In this section, we state two identification results. The first one identifies the restricted marginal treatment effect function using only Assumptions 1-5 and a regularity condition. Its proof is a direct consequence of the Proposition 3.1 and the relationship between distribution functions and expected values of non-negative variables. The second result identifies the average marginal treatment effect function after imposing additional support restrictions.

Corollary H.1. *Suppose that Assumptions 1-5 and Assumption B.7 listed in the Appendix B hold. Then, the $RMTE(v)$ function (Equation (H.4)) is point-identified for any $v \in \mathcal{P}$.*

Notice that, under Assumptions 1-5, we cannot point-identify the MTE function (Equation (H.2)). The rationale for this "negative" result is that we may never observe realizations of Y^* beyond γ_C when the support of C is smaller than the support of Y^*. In those cases, we cannot identify the right-tail of the distributional marginal treatment response, i.e., we cannot identify $DMTR_d(y,v)$ for $y \geqslant \gamma_C$. Of course, when the support of C is contained in the support of $Y^*(d)$, this situation does not arise, and we can identify the MTE function as long as it is well-defined. This is precisely what Assumptions H.1 and H.2 impose. We summarize this result in the next corollary.

Corollary H.2. *Suppose that Assumptions 1-H.2 and Assumption B.7 listed in the Appendix B hold. Then, $MTE(v)$ is point-identified for any $v \in \mathcal{P}$.*

H.4   Estimation and Inference

In this section, we provide algorithms on how to semiparametrically estimate the MTE and RMTE functions based on the identification results described in Corollary H.2. We also provide practical methods to conduct point-wise inference around these two target parameters.

Using the $DMTE$ estimator described in Algorithm 4.2, we can estimate the $RMTE(v,x)$ function according to

$$\widehat{RMTE}(v,x) \;=\; -\int_0^{\gamma_C} \widehat{DMTE}(y,v,x)dy. \tag{H.5}$$

Importantly, this estimator is asymptotically normal as formalized by Theorem H.1

Theorem H.1. *Suppose that Assumptions 1-5 and Assumptions B.1-B.7 listed in Appendix B hold. Then, as $n \to \infty$, for each fixed $v \in \mathcal{P}$, and $x \in \mathcal{X}$,*

$$\sqrt{n}\left(\widehat{RMTE}(v,x) - RMTE(v,x)\right) \xrightarrow{d} N(0, V_{v,x}^{rmte}),$$

with $\widehat{RMTE}(v,x)$ as defined in Equation (H.5) and $V_{v,x}^{rmte}$ as defined in Appendix B.

Although Theorem H.1 indicates that one can potentially conduct inference using plug-in estimates of the variance, this procedure would involve estimating additional nuisance functions

and could be cumbersome in practice. To avoid this issue, we propose using a weighted bootstrap procedure as in Ma and Kosorok (2005b) and Chen and Pouzo (2009). This bootstrap procedure is very straightforward to implement, as described in the next algorithm.

Algorithm H.1 (Weighted-Bootstrap Implementation)**.**

1. *Estimate DMTE and RMTE according to Algorithm 4.2 and Equation* (H.5)*.*

2. *Generate* $\{\omega_i, i = 1, \ldots, n\}$ *as a sequence of independent and identically distributed non-negative random variables with mean one, variance one, and finite third moment (e.g.,* $\omega_i \sim Exp(1)$*).*

3. *Compute the propensity score coefficients associated with Equation* (4.1) *by minimizing the weighted least squares function, i.e,*

$$\hat{\theta}^{fs,*} = \underset{\theta^{fs} \in \Theta^{fs}}{\arg\min} \; n^{-1} \sum_{i=1}^{n} \omega_i \left(D_i - \alpha_0 - X_i'\alpha_X - C_i\alpha_C - \psi^L(Z_i)'\alpha_Z, \right)^2 \qquad \text{(H.6)}$$

   where $\hat{\theta}^{fs,*} = (\hat{\alpha}_0^*, \hat{\alpha}_X^{*\prime}, \hat{\alpha}_C^*, \hat{\alpha}_Z^*))'$. *Denote its trimmed fitted propensity score values by* $\hat{P}_i^*$ *as defined in Equation* (4.4)*, but with* $\hat{\theta}^{fs,*}$ *in place of* $\hat{\theta}^{fs}$*.*

4. *Consider the same grid of values for the duration outcome* Y *as defined in Step 2 of Algorithm 4.2.*

5. *For each* $k \in \{0, \ldots, K\}$ *and each* $d \in \{0, 1\}$*, estimate the conditional distribution function of* $Y \cdot \mathbf{1}\{D = d\}$ *given* $P(Z, C)$*,* C*, and* X *using the distribution regression model (Equation* (4.5)*) with estimated coefficients*

$$\hat{\theta}^* (y, d) = \underset{\theta \in \Theta}{\arg\max} \; \frac{1}{n} \sum_{i=1}^{n} \omega_i \; \ln \ell_\theta(\mathbf{1}\{Y_i \leqslant y, D_i = d\}, X_i, C_i, \hat{P}_i^*; y, d). \qquad \text{(H.7)}$$

6. *Follow Steps 4-9 of Algorithm 4.2 and Equation* (H.5) *using* $\hat{\theta}^* (y, d)$ *instead of* $\hat{\theta}(y, d)$*. Denote by* $\widehat{DMTE}^* (y_k, v, x)$ *and* $\widehat{RMTE}^* (v, x)$ *the distributional and restricted marginal treatment effects estimates.*

7. *Repeat Steps 2-6 B times, e.g.,* $B = 399$*, and collect* $\left\{ \left(\widehat{DMTE}^* (y_k, v, x) \right)_b, b = 1 \ldots, B \right\}$*. Do the same for the* $\widehat{RMTE}^* (v, x)$*.*

8. *Obtain the* $(1 - \alpha)$ *quantile of* $\left\{ \left| \left(\widehat{DMTE}^* (y_k, v, x) - \widehat{DMTE}(y_k, v, x) \right)_b \right|, b = 1 \ldots, B \right\}$*,* $c^{dmte,*}(y_k, v, x; \alpha)$*. Compute the analogous critical values based on* $\widehat{RMTE}^* (v, x)$*.*

9. *Construct the* $1 - \alpha$ *(pointwise) confidence interval for* $DMTE(y_k, v, x)$ *as* $\hat{C}^{dmte}(y_k, v, x) = [\widehat{DMTE}(y_k, v, x) \pm c^{dmte,*}(y_k, v, x; \alpha)]$*. Define* $\hat{C}^{rmte}(v, x; \alpha)$ *analogously.*

The next theorem establishes that the above bootstrap procedure has asymptotically correct coverage.

Theorem H.2. *Under the assumptions of Theorem H.1, for any* $0 < \alpha < 1$*, and for each* $v \in \mathcal{P}$*,* $x \in \mathcal{X}$*,* $y < \gamma_C$*, and* $\tau \in (0, \bar{\tau}(v, x))$*, for* $n \to \infty$*,* $\mathbb{P}\left(RMTE(v, x) \in \hat{C}^{rmte}(v, x; \alpha) \right) \to 1 - \alpha$*.*

Note that the functionals in Theorems H.1 and H.2 provide a covariate-specific treatment effect. In our application's context, we can get court-district-specific RMTE estimates of the effect of fines and community service sentences on time-to-recidivism. However, it may be desirable to further aggregate the MTE functionals as a way to summarize these effects.

As explained at the end of Section 4.2, we decided to aggregate the court-district-specific DMTE functionals across court districts using the proportion of cases per court district as weights. Analogously, let $RMTE^{avg}(v) = -\int_0^{\gamma_C} DMTE^{avg}(y, v) dy$ be the average restricted marginal treatment effect. This functional can be straightforwardly estimated using functionals of

$$\widehat{DMTR}_d^{avg}(y, v) = \sum_{x \in \mathcal{X}} \hat{w}_x \ \widehat{DMTR}_d(y, v, x),$$

with $\widehat{DMTR}_d(y, v, x)$ as in Equation (4.10), just like in Equations (4.11) and (H.5). Their large-sample properties follow from the delta method and are summarized in the following corollary.

Corollary H.3. *Suppose that Assumptions 1-5 and Assumptions B.1-B.7 listed in Appendix B hold. Then, as $n \to \infty$, for each fixed $v \in \mathcal{P}$,*

$$\sqrt{n} \left(\widehat{RMTE}^{avg}(v) - RMTE^{avg}(v) \right) \xrightarrow{d} N(0, V_v^{rmte,avg}).$$

It is also straightforward to construct a weighted-bootstrap confidence interval for these functionals by using $\hat{w}_x^* = n^{-1} \sum_{i=1}^n \omega_i \ \mathbf{1}\{X_i = x\}$ as weights for the MTE functionals. We omit a detailed description to avoid repetition.

H.5   Empirical Results

H.5.1   Assessing the plausibility of our assumptions

The interpretation of the RMTE function relies strongly on Assumption H.2. If such an assumption is not plausible, we must interpret our results accordingly and avoid jumping from restricted MTE functionals to overall MTE ones.

In this subsection, we use descriptive statistics to assess the plausibility of our support restriction.[34] To do so, we analyze how the conditional share of no recidivism varies with the censoring variable. More precisely, Figure H.1 shows the probability that a defendant does not recidivate during our sampling period, given the value of her censoring variable. Conditioning on the defendants who stay the longest in our sample (large values of C), we still find a 30% probability that they do not recidivate during the observation period. This result suggests that our support restriction in Assumption H.2 is not valid in this context, implying that we should

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[34]The credibility of our other identifying assumption is discussed in Section 5.1.

interpret our restricted marginal treatment effect estimates accordingly. In any event, this result does not invalidate our DMTE and QMTE procedures since they rely on Assumptions 1-5 only. It just stresses that censoring is an important complication we must address.

Figure H.1: Probability of No Recidivism during the Sampling Period: $\mathbb{P}\left[Y^* > C \mid C\right]$
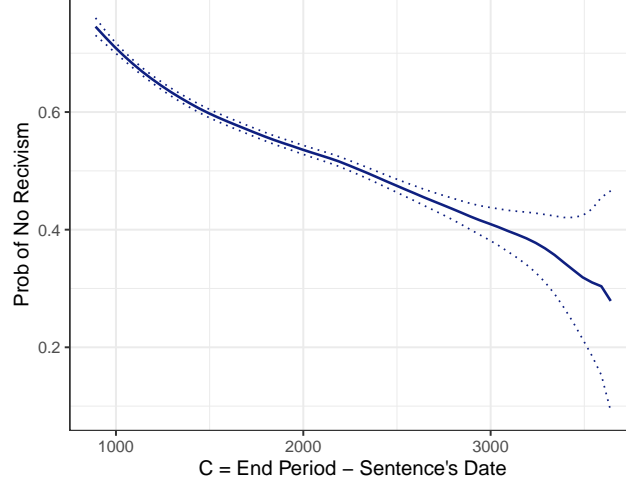
Notes: Figure H.1 shows the probability that a defendant does not recidivate during our sampling period given the value of her censoring variable. This nonparametric function was estimated using a local linear regression with an Epanechnikov kernel based on Calonico et al. (2019). The bandwidth was optimally selected according to the IMSE criterion. The dotted lines are robust bias-corrected 95% confidence intervals.
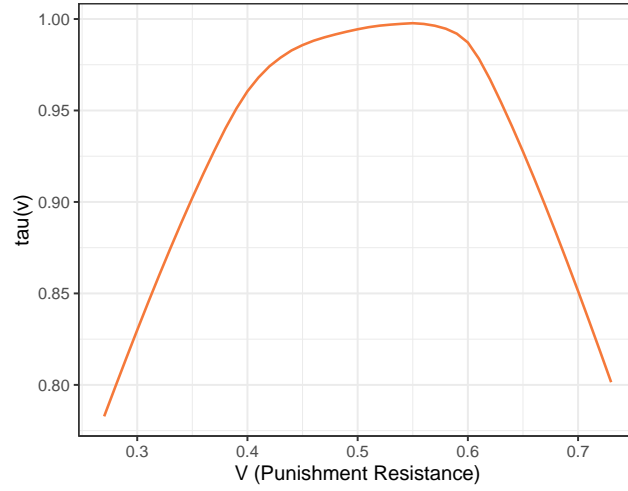
H.5.2 Estimated RMTE function

To estimate the RMTE functions in our empirical application, we flexibly account for court district fixed effects. More precisely, we estimate 193 district-specific functions for each of our treatment effect parameters (Theorem H.1). Although very flexible, this strategy makes it challenging to concisely report a summary result. The way we proceeded was to average these district-specific functions over court districts using the proportion of cases per court district as weights, as in Corollary H.3. In this section, we report the average RMTE function and compare our proposed methods against standard methods in the literature.

The main advantage of the $RMTE\left(\cdot\right)$ in comparison to the $DMTE$ and $QMTE$ functions is its ability to summarize all results in one single function. Figure H.3 plots the point estimates of this function in purple. Before discussing this result, we must understand how restricted is the RMTE function (Equations (H.3) and (H.4)) compared to the overall MTE function. If the support of C is "too small" compared to the support of the time-to-event outcome, RMTE may be further away from the MTE function, affecting its interpretability.

Figure H.2 plots the estimated maximum identifiable quantile: $\overline{\tau}\left(v\right) := \min\left\{\overline{\tau}_0\left(v\right), \overline{\tau}_1\left(v\right)\right\}$ where $\overline{\tau}_d\left(v\right) := DMTR_d\left(\gamma_C, v\right)$ for any $d \in \{0, 1\}$ and $\gamma_C := \inf\left\{c \in \overline{\mathbb{R}}: \mathbb{P}\left[C \leqslant c\right] = 1\right\}$. If the maximum identifiable quantile, $\overline{\tau}\left(\cdot\right)$ is "far away" from 1, then the support of C is "small"

compared to the support of time-to-event outcome.

Figure H.2: Maximum Identifiable Quantile: $\overline{\tau}(v)$

Notes: The orange line plots the estimated maximum identifiable quantile, $\overline{\tau}(v)$, for each value of the unobserved resistance to treatment. The definition of $\overline{\tau}(v)$ can be found in Corollary 3.1.

Figure H.2 shows that the RMTE function is almost an unrestricted mean for $v \in (.5, .6)$. However, the censoring problem is binding for small and large values of the unobserved resistance to treatment. Consequently, the RMTE is further away from the MTE function for extreme values of punishment resistance.

Now, analyzing the point estimates of the RMTE function (purple line in Figure H.3), we find that the estimated restricted average marginal treatment effects decrease with the unobserved resistance to treatment. This result is statistically significant at the 10% significance level according to Figure D.5d). This result is similar to the QMTE results in Section 5.2.2.2.

Furthermore, Figure H.3 compares our proposed methods against other available methods in the literature. Differently from our approach, these estimates ignore that the outcome variable is right-censored and provide different conclusions when compared against our proposed estimator. The light blue line denotes a "naive" version of our estimator that follows the same steps as described in Section 4.2 but does not condition on the censoring variable. The orange line is the treatment coefficient of a two-stage least squares (2SLS) regression that uses the censored outcome variable as the left-hand side variable, controls for court district fixed effects, and uses the judge's punishment rate as the instrument for the defendant being punished. The dark blue line is the estimated average MTE function based on a parametric estimator (Cornelissen et al., 2016, Appendix B.2) that imposes a linear MTE curve and ignores censoring concerns by directly using the level of the censored outcome variable.

Figure H.3 highlights three interesting results. First, the standard MTE method's estimates (dark blue line) tend to be larger than the estimated $RMTE$ function that accounts for censoring (purple line). Second, the naive estimator (light blue line) finds a $RMTE$ function that is

Figure H.3: RMTE against Standard Methods
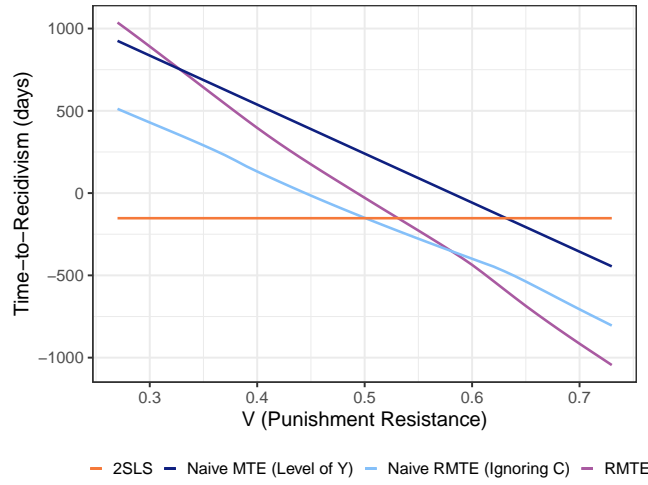
Notes: The purple line shows the average RMTE function (Corollary 4.1). The light blue line denotes a naive version of our estimators that ignores censoring. The orange line is the treatment coefficient of a 2SLS regression. The dark blue line is the estimated average MTE function based on a parametric estimator (Cornelissen, Dustmann, Raute and Schonberg, 2016, Appendix B.2) that directly uses the level of the censored outcome variable.

less steep. Importantly, both blue lines do not lie entirely within the 90%-confidence intervals of the estimated $RMTE$ function that accounts for censoring (Figure D.5d). Third, the 2SLS estimate (orange line) finds a negative effect of punishment on time-to-recidivism, suggesting that punishing defendants with fines and community service has led to faster recidivism. This 2SLS result does not capture that punishing defendants with alternative sentences may increase time-to-recidivism for some defendant types, as suggested by our preferred $RMTE$ estimate (purple line). Importantly, the 2SLS estimate does not lie entirely within the 90%-confidence intervals of the estimated $RMTE$ function (Figure D.5d).

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