

Was Javert right to be suspicious?

Unpacking treatment effect heterogeneity of alternative sentences on time-to-recidivism in Brazil *

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Abstract

This paper presents new econometric tools to unpack the treatment effect heterogeneity of punishing misdemeanor offenses on time-to-recidivism. We show how one can identify, estimate, and make inferences on the distributional, quantile, and average marginal treatment effects in setups where the treatment selection is endogenous and the outcome of interest, usually a duration variable, is potentially right-censored. We explore our proposed econometric methodology to evaluate the effect of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, between 2010 and 2019, leveraging the as-if random assignment of judges to cases. Our results highlight substantial treatment effect heterogeneity that other tools are not meant to capture. For instance, we find that people who most judges would punish take longer to recidivate as a consequence of the punishment, while people who would be punished only by strict judges recidivate at an earlier date than if they were not punished. This result suggests that designing sentencing guidelines that encourage strict judges to become more lenient could reduce recidivism.

Keywords: Duration Outcomes, Instrumental Variable, Alternative Sentences, Recidivism

JEL Codes: C24, C31, C36, C41, K42

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To owe his life to a malefactor, to accept that debt and to repay it; to be, in spite of himself, on a level with a fugitive from justice, and to repay his service with another service; to allow it to be said to him, “Go,” and to say to the latter in his turn: “Be free”; to sacrifice to personal motives duty, that general obligation, and to be conscious, in those personal motives, of something that was also general, and, perchance, superior, to betray society in order to remain true to his conscience; that all these absurdities should be realized and should accumulate upon him,—this was what overwhelmed him.

Les Misérables by Victor Hugo

1 Introduction

Understanding how different types of sanctions impact the behavior of defendants is a critical area of research in the field of Economics of Crime. For misdemeanors, which are relatively minor offenses, we know relatively little about the causal effects of prosecution on defendants’ subsequent criminal justice involvement (Agan, Doleac and Harvey, 2023), and arguably even less about the effect of alternative sentences on defendants’ recidivism.¹ This is a particularly important topic, as a misdemeanor charge is often the point of entry of individuals to the criminal justice system. If they are convicted, they will then acquire a criminal record. This could “lower the cost” of committing other crimes, or work as intended and prevent future criminal behavior. In practice, it is unclear which direction dominates, and it is likely that this varies from individual to individual. Being able to understand the types of defendants that are on either side is therefore desirable and policy-relevant.

In this article, we propose econometric tools that are tailored to highlight treatment effect heterogeneity with respect to the unobserved punishment resistance on time-to-recidivism. These tools can then be used to shed light on to whom punishments are working as intended in terms of avoiding (or postponing) recidivism. Importantly, our tools account for the fact that (i) time-to-recidivism is a duration outcome that is subject to right-censoring, i.e., not all defendants recidivate by the end of the sampling period (but may do it later on); (ii) treatment selection is endogenous and judges are likely to have more information about the case than researchers; (iii) individuals may be inherently heterogeneous (essential heterogeneity); (iv) one may be interested in causal effects beyond local average treatment effect parameters; (iv) distributional features of time-to-recidivism may also be relevant.

We achieve these goals by extending the marginal treatment effects (MTE) framework developed by Heckman and Vytlacil (1999, 2005) to setups in which the outcome variable is

¹See Huttunen, Kaila and Nix (2020), Giles (2021), Klaassen (2021), Possebom (2022), and Lieberman, Luh and Mueller-Smith (2023) for some advances in this area.

right-censored. The main requirement to use our tools is having access to a continuous instrument such that the propensity score has large support.² In the context of crime economics, this instrument is usually given by the trial judge’s leniency rate. Our tools can be used to recover average, distributional, and quantile MTE (Carneiro and Lee, 2009).

In our view, the MTE framework is particularly attractive to studying the effect of punishments on time-to-recidivism. For example, it allows one to assess the treatment effect of punishment on recidivism for defendants on a margin of indifference between being punished or not. By considering different degrees of unobserved punishment resistance, the MTE provides a detailed picture of how punishments heterogeneously affect recidivism and can be used to design better sentencing criteria and/or train judges to follow a specific protocol. For example, suppose that one finds a negatively sloped MTE function with some positive and negative effects. This would suggest that defendants who would be punished even by very lenient judges — i.e., defendants with low unobserved punishment resistance — would take more time to recidivate as a result of the punishment (punishment is working as intended). On the other hand, defendants who would be fined only by very strict judges — i.e., defendants with high unobserved punishment resistance — would recidivate sooner than if they were not punished (punishment is not effective, perhaps because of scaring effects of a criminal record). Such degree of heterogeneity is usually washed out when using single summaries of treatment effects such as local average treatment effect (LATE) (Imbens and Angrist, 1994). However, even when one is interested in summary measures of causal effects, one can use the MTE function to construct them (Heckman and Vytlačil, 2005; Heckman, Urzua and Vytlačil, 2006). It is also interesting to mention that exploring a continuous instrument makes the definition of “complier” less clear than in the binary instrument case, which could potentially make the LATE results harder to interpret formally. The MTE does not focus on “compliers”, so it avoids this potential limitation.

Dealing with time-to-recidivism, or more generally, a duration variable that is subject to right-censoring, introduces some interesting challenges depending on the censoring mechanism. For instance, if censoring is independent of potential outcomes, we are able to point-identify the distributional marginal treatment effect (DMTE) and quantile marginal treatment effect (QMTE) functions for some but not necessarily all distribution support points or quantiles. Nonparametrically identifying the entire DMTE and QMTE functions, which are required to identify the average MTE (henceforth MTE for simplicity), is only possible if the support of the censoring variable is at least as large as the support of the duration outcome. When this restriction is not satisfied, one can only nonparametrically point-identify truncated MTE functions. Addressing these challenges, we propose semiparametric estimators and inference

²See Brinch, Mogstad and Wiswall (2017) and Mogstad, Santos and Torgovitsky (2018) for extensions of the MTE framework that does not require this support condition.

procedures for the DMTE, QMTE, and (truncated) MTE functions and establish their large sample properties.

Now, if censoring is potentially dependent on the potential outcomes, point-identification of DMTE and QMTE functions is not feasible without additional assumptions and data requirements. In such cases, we provide alternative assumptions that ensure partial identification of the target parameters. First, we consider the restriction that defendants are committing fewer crimes over time, which is implied by a negative regression dependence between potential outcomes and censoring variables (Lehmann, 1966). Second, we discuss a continuous relaxation of the independent censoring assumption.

In some setups, to bypass the challenges associated with right censored time-to-recidivism, researchers may focus on recidivism within a given time frame, say two years. Although this is convenient and generically valid, the choice of cutoff is arbitrary, and it may be the case that punishment has no effect on recidivism within two years but then has an effect within two years and a half or within one year.³ One can interpret our DMTE results as an extension of this “binarization” approach that aims to avoid choosing arbitrary cutoffs and, instead, consider recidivism within y periods for a continuum of $y \in \mathbb{R}_+$. Our QMTE and MTE results “transform” our DMTE results so the underlying treatment effects are expressed in the same units as the time-to-recidivism outcome, which can lead to additional insights. Furthermore, when a policymaker is interested in minimizing the cost of recidivism inter-temporally, they may discount the cost of recidivism more strongly if the time-to-recidivism is longer. Therefore, to make more informed treatment allocations (or recommendations), the policymaker needs information on time-to-recidivism beyond whether or not a defendant recidivates within two years (see Appendix E.1 for a discussion). In such cases, however, one needs to tackle the censoring problem directly. Failing to do so may lead to misleading conclusions.

We show how our causal inference tools can be used in practice by evaluating the effect of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, between 2010 and 2019.⁴ Our treated group (punished group) is the defendants who were fined or sentenced to community services, and our untreated group (unpunished group) contains defendants who were acquitted or whose cases were dismissed. To measure recidivism, we check whether the defendant’s name appears in any criminal case within the sample period after the final sentence’s date. More precisely, our outcome variable is the time between the final sentence and a subsequent criminal case. Since the sampling period

³In Appendix E.2, a simple example illustrates that focusing on quantile and average treatment effects for duration outcomes may provide different conclusions than focusing on short-run recidivism indicators.

⁴São Paulo is the largest state in Brazil, with a population above 44 million people according to the Brazilian Census in 2022. Moreover, analyzing the impact of judicial policies on criminal behavior in this state is relevant due to its relatively high criminality. For example, according to São Paulo Public Safety Secretary, there were 6.48 murders, 878.83 thefts, and 490.23 robberies per 100,000 inhabitants in 2020. Importantly, theft is one of the most common crimes in our sample.

is finite, the outcome variable is right-censored.

To deploy our proposed methodology, we need a continuous instrumental variable since we do face endogenous selection into punishment. We use the trial judge’s leave-one-out rate of punishment (or “leniency rate”) as an instrument for the trial judge’s decision (Bhuller, Dahl, Loken and Mogstad, 2019; Agan et al., 2023). Importantly, this instrumental variable is continuous with large support, and is independent of the defendant’s counterfactual criminal behavior because judges are randomly assigned to cases conditional on court districts according to state law in São Paulo. Our outcome data — time-to-recidivism — is right-censored by construction, requiring a methodology that accounts for this identification challenge.

We find that the QMTE functions for .10, .15, .25, .40, .50 and .75 quantiles and the restricted MTE function averaged across all court districts are heterogeneous with respect to unobserved punishment resistance: the treatment effects being sometimes positive and sometimes negative. More precisely, we find that people who would be punished by most judges (those with low punishment resistance) take longer to recidivate as a consequence of the punishment, while people who would be punished only by strict judges (high punishment resistance) recidivate at an earlier date than if they were not punished. This result suggests that designing sentencing guidelines that encourage strict judges to become more lenient could increase time-to-recidivism.

We also compare our results with methods that ignore the time-to-recidivism being right-censored. In particular, we find that using a linear MTE estimator exacerbates the treatment effects across the unobserved punishment resistance variable. We find the same issue when ignoring the censoring problem and estimating the MTE model semiparametrically. If one were to use two-stage least squares or IV quantile regressions (ignoring censoring), one would find that treatment effects are slightly negative but would not be able to highlight heterogeneity as in the QMTE and RMTE functions. These differences highlight that our tools can indeed bring new insights to policy discussions.

Related literature: This article contributes to different branches of literature. Concerning its theoretical contribution, our work contributes to the literature on MTE by extending the MTE framework of Heckman and Vytlačil (1999, 2005), Heckman et al. (2006), and Carneiro and Lee (2009) to a setting with right-censored data.⁵ We also contribute to the literature on duration outcomes; see, e.g., Khan and Tamer (2009), Frandsen (2015), Tchetgen, Walter, Vansteelandt, Martinussen and Glymour (2015), Sant’Anna (2016, 2021), Beyhum, Florens and Keilegom (2022), Delgado, Garcia-Suaza and Sant’Anna (2022). None of these papers consider MTE-type parameters as we do. Among these, the closest work to ours is Frandsen (2015), which considers the case where the censoring variable is observed and shows how one can identify

⁵The MTE framework has also been extended to settings with sample selection (Bartalotti, Kedagni and Possebom, 2022), misclassified treatment variables (Acerenza, Ban and Kédagni, 2021; Possebom, 2022), discrete instrumental variables (Brinch et al., 2017; Mogstad et al., 2018; Acerenza, 2022), and possibly invalid instruments (Mourifie and Wan, 2020).

distributional and quantile local treatment effects, assuming that censoring is exogenous. Our results can be interpreted as an extension of Frandsen (2015) to the MTE framework, possibly allowing for endogenous censoring.

Concerning its empirical contribution, our work is inserted in the literature about the effect of fines and community service sentences on future criminal behavior; see, e.g., Huttunen et al. (2020), Giles (2021), Klaassen (2021), Possebom (2022), and Lieberman et al. (2023). They all focus on binary variables indicating recidivism within a pre-specified period. Within these, as we build on his dataset, Possebom (2022) is the closest to ours. However, his focus is very different from ours, and he does not handle duration outcomes as we do.

This paper is organized as follows. Section 2 describes the data and explains why focusing on long-term recidivism is useful in our empirical application. Section 3 presents our structural model, discusses our identifying assumptions and provides our identification results with a right-censored outcome variable. Moreover, Section 4 explains how to semiparametrically estimate the objects that are necessary to implement the identification strategy described in the previous section. Finally, Section 5 discusses the empirical results, while Section 6 concludes.

This paper also contains an online supporting appendix. Our main identification proofs are detailed in Appendix A. Appendix B derives the asymptotic distribution of our semiparametric estimators. In Appendix C, we summarize how we constructed our dataset. Additional empirical results can be found in Appendix D. Appendix E provides two arguments that justify focusing on the MTE function of duration outcomes. Appendix F identifies a conditional version of our target parameters under weaker assumptions than the ones used in the main text. Finally, Appendix G proposes alternative partial identification strategies.

2 Empirical Context, Data, and Target Parameters

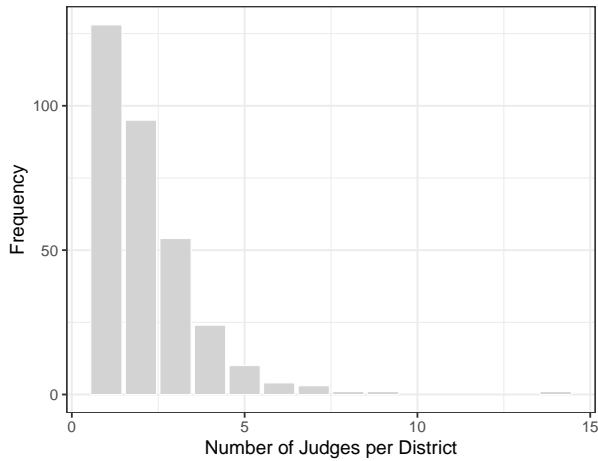
We study the effect of alternative sentences in the form of fines and community service on time-to-recidivism in the state of São Paulo, Brazil. Towards this end, we collect data from all criminal cases brought to the Justice Court System in the State of São Paulo, Brazil, between January 4th, 2010, and December 3rd, 2019.⁶ According to Brazilian law, criminal charges whose maximum prison sentence is less than four years must be punished with a fine or a community service sentence if the defendant is found guilty. As we are particularly interested in the effect of these alternative sentences, we focus on these specific criminal cases. We also restrict our sample to cases that started between 2010 and 2017 to ensure that every defendant is followed up for at least two years after their case is brought to trial. Based on these restrictions, the most common types of crime in our sample are theft and domestic violence.

⁶See Appendix C for an overview of the data-construction.

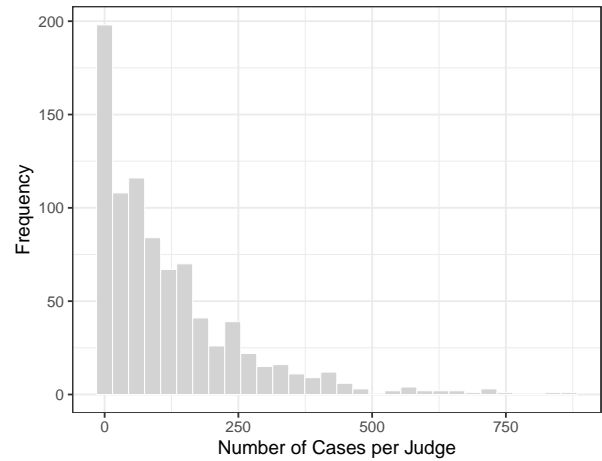
There are 332 court districts in the state of São Paulo. Criminal complaints are analyzed by a trial judge working at the court with geographic jurisdiction over the location of the alleged offense. Moreover, there are 862 trial judges during our sample period. Out of these, we keep 642 judges who analyzed more than 20 cases. In court districts that have more than one judge, the case is randomly allocated to one of the judges. Out of the 332 court districts in our sample, 193 have more than one judge who analyzed more than 20 cases. Given that our econometric procedure explores the random allocation of judges to criminal cases and their different leniency levels, we restrict our attention to court districts with more than two judges who analyzed 20 cases or more. After imposing these two restrictions, our sample has 525 trial judges.

Figure 1 shows the distribution of the number of judges for every court district in São Paulo (Subfigure 1a) and the distribution of the number of cases for every judge in São Paulo (Subfigure 1b). The average number of cases per judge is 120 in our full sample and 159 after we focus on judges who analyzed more than 20 cases. Furthermore, the average number of judges per court district is 2.2 judges in full sample and 3.0 judges in our restricted sample.

Figure 1: Descriptive Statistics for the Number of Judges and the Number of Cases



(a) Histogram of the Number of Judges per Court District



(b) Histogram of the Number of Cases per Trial Judge

Notes: Subfigure 1a plots the histogram of the number of judges per court district in our full sample, while Subfigure 1b plots the histogram of the number of cases per trial judge in our full sample.

2.1 Defining the variables of interest

In our dataset, we observe the defendant's full name, the defendant's court district, the case's starting date, the assigned trial judge's full name, the case's final ruling, and the case's final ruling's date. All our variables of interest will be constructed from these pieces of information. Henceforth, let X denote the full set of court district dummies, which will play the role of

covariates in our analysis.

Let us start with our treatment variable, D , which denotes the final ruling in the case. Defendants who were fined or sentenced to community services because they were either convicted or signed a non-prosecution agreement according to the final ruling in their case belong to our treatment group, $D = 1$. Defendants who were acquitted or their cases were dismissed according to the final ruling in their case belong to our comparison group, $D = 0$.

In this article, our outcome of interest, Y^* , is the “time-to-recidivism”, i.e., the number of days it takes for a defendant to appear in court once again after the case’s final ruling’s date. Here, note that our outcome of interest is a duration variable and that some defendants may not recidivate by the end of our sampling period, though they may recidivate later. Putting it simply, we do not always observe Y^* , but rather observe a right-censored version of Y^* , $Y = \min(Y^*, C)$, where C is a right-censoring variable. In our context, C is the follow-up period for each defendant, i.e., the number of days from their case’s final ruling date to December 3rd, 2019.

Apart from the censoring problem, it is important to be explicit about how we define recidivism. In this paper, a defendant i in a case j recidivated by the end of our sample if and only if defendant i ’s full name appears in a case \bar{j} whose starting date is after case j ’s final sentence’s date.⁷ Then, we measure our outcome variable as the number of days between case j ’s final ruling’s date and case \bar{j} ’s starting date.⁸ If defendant i did not recidivate by the end of the sampling period, then $Y = C$.

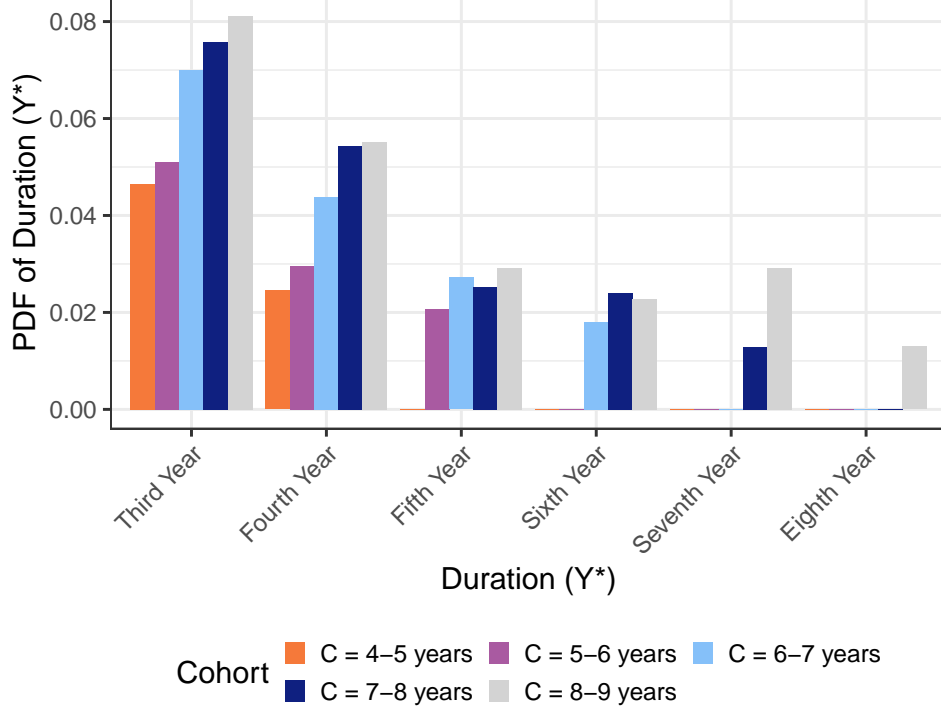
At this stage, it is important to stress that we are not adopting a more restrictive notion of “short-run” recidivism based on a fixed period, say two years, which could potentially allow us to “ignore” the censoring problem. Instead, we decide to focus on time-to-recidivism directly, which, in our view, entails some important advantages. For instance, we do not need to pick a threshold to define (short-run) recidivism arbitrarily. Doing so may lead to potentially sensitive conclusions, as illustrated in an example in Appendix E.2. If almost all defendants who recidivate do it in the short run, then focusing on short-run measures would be sufficient. However, this is an empirical matter and should be handled as so. To assess if this is the case in our data, Figure 2 displays estimates of the right tail of the probability mass function (PDF) of the uncensored potential outcome (Y^*) among cohorts defined based on the censoring variable. These descriptive results reveal that, in the case of the state of São Paulo, a non-negligible share of defendants have their first recidivism event in their fifth, sixth, or seventh year after their sentence’s date, implying that analyzing long-term recidivism is relevant and that tackling the

⁷To match defendants’ names across cases, we follow the same procedure as in Possebom (2022) and define a fuzzy match if the similarity between full names in two different cases is greater than or equal to 0.95 using the Jaro–Winkler similarity metric.

⁸Case \bar{j} can be about any type of crime, including more severe crimes with a maximum sentence of over four years, while case j has to be about a crime whose a maximum sentence is at most four years.

censoring problem directly is important. See Appendix E.1 for additional motivations from a welfare maximizer decision-maker perspective.

Figure 2: PDF of the Uncensored Outcome given the Defendant’s Cohort



Notes: This figure shows the right tail of the probability mass function of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable, $\mathbb{P}[y_1 \leq Y^* \leq y_2 | C]$ where y_1 and y_2 define the bins indicated in the x-axis. In particular, these conditional PDFs are evaluated at six bins of the uncensored potential outcome. For example, “Third Year” denotes that the first recidivism event occurred between 730 days ($= y_1$) and 1095 days ($= y_2$), while “Fourth year” denotes that the first recidivism event occurred between 1095 days ($= y_1$) and 1460 days ($= y_2$). Each color denotes a different cohort: orange denotes defendants who are observed for at least four years and at most five years during our sampling period, purple denotes defendants who are observed for at least five years and at most six years, light blue denotes defendants who are observed for at least six years and at most seven years, dark blue denotes defendants who are observed for at least seven years and at most eight years, and gray denotes defendants who are observed for at least eight years and at most nine years. The y-axis denotes the value of the PDF.

As it will be clear in the next sections, our causal inference procedures leverage the availability of an instrumental variable Z with large support. In our context, the instrument Z is the trial judge’s leniency rate. This variable equals the leave-one-out rate of punishment for each trial judge, where the defendant’s own decision is excluded from this average. We ensure that the minimum and maximum values of the Z are the same across both treatment arms to enforce better overlap properties.

Our final sample has 43,468 case-defendant pairs, and some summary statistics are presented in Table 1. It shows the outcome’s mean, 1st decile, 1st quartile and median for all defendants, for the defendants who were punished (treated group), and for the defendants who were not punished (control group). It also shows the sample size of each one of these three groups. The comparison between the treated and control groups suggests that being punished slightly

harms defendants. However, this naive comparison ignores endogenous selection-into-treatment, right-censoring, and heterogeneous treatment effects. As so, one should be careful with such a comparison, as these descriptive statistics may not have a causal interpretation.

Table 1: Descriptive Statistics — Outcome Variable

	Unconditional	Treated Group	Control Group
Mean	1,081	1,047	1,116
1 st Decile	77	69	86
1 st Quartile	364	321	430
Median	1082	1047	1127
Number of Observations	43,468	22,060	21,408

Note: The treated group receives a punishment, i.e., its defendants were fined or sentenced to community services because they were either convicted or signed a non-prosecution agreement. The control group did not receive a punishment, i.e., its defendants were acquitted or its cases were dismissed. The outcome variable measures the number of days between the case’s final ruling’s date and the first recidivism event if the defendant recidivates or the number of days between the case’s final ruling’s date and the end of the sampling period if the defendant did not recidivate. An observation is a case-defendant pair.

2.2 Causal Questions of Interest

For each defendant i , let $Y_i^*(1)$ be the potential time-to-recidivism if defendant i were punished with a fine or community service, and let $Y_i^*(0)$ be the potential time-to-recidivism if defendant i were not punished with a fine or community service. Defendant i ’s treatment effect is therefore $\theta_i = Y_i^*(1) - Y_i^*(0)$. Ideally, we would like to learn θ_i for all defendants, though that is very challenging (if not impossible) when we allow for (a) heterogeneous treatment effects across defendants and (b) whether a defendant is punished or not being related to θ_i (“Essential Heterogeneity” as in Heckman et al., 2006).

Due to these challenges, it is common for researchers to focus on aggregated summary measures of θ_i , such as the average treatment effect among “compliers” (Imbens and Angrist, 1994), defined as $LATE = \mathbb{E}[Y^*(1) - Y^*(0)|\text{Compliers}]$ (see, e.g., Agan et al., 2023; Bhuller et al., 2019; Huttunen et al., 2020).⁹ Although interesting and policy-oriented, such aggregated measures of causal effects are unsuitable for highlighting important types of treatment effect heterogeneity. In particular, these parameters cannot answer questions related to whether defendants with high punishment resistance (i.e., defendants who would only be punished by very strict judges) would take longer to recidivate if they were punished. The same goes for defendants with lower punishment resistance. These are the key causal questions we are trying to answer in this paper. More specifically, we want to provide a more detailed picture of how alternative punishments heterogeneously affect recidivism with respect to the defendant’s (unobserved)

⁹As discussed before, these papers use a different outcome of interest Y^* that bypass the censoring issues we face, but, we can ignore these issues while discussing our causal questions of interest (as this does not play a prominent role on it).

punishment resistance, which we denote by V . Here, punishment resistance may capture the evidence gathered against the defendant and additional defendant-specific characteristics.

One can measure the causal effect of fines and community services on time-to-recidivism for defendants with a given punishment resistance using the notion of distribution, quantile, and average treatment effects. We cover them all, as they can be used to answer complementary policy-relevant questions (Heckman and Vytlacil, 2005, Carneiro and Lee, 2009, Carneiro, Heckman and Vytlacil, 2011). All these causal parameters build on the Marginal Treatment Effects framework of Heckman and Vytlacil (2005). We now carefully define these and highlight how one can interpret them.

For $d \in \{0, 1\}$, let the distributional, quantile, and average marginal treatment response functions be defined as

$$DMTR_d(y, v) := \mathbb{P}[Y^*(d) \leq y | V = v], \quad (2.1)$$

$$QMTR_d(\tau, v) := \inf\{y \in \mathbb{R}_+ : \mathbb{P}[Y^*(d) \leq y | V = v] \geq \tau\}, \quad (2.2)$$

$$AMTR_d(v) := \mathbb{E}[Y^*(d) | V = v], \quad (2.3)$$

where $y \in \mathbb{R}_+$, $\tau \in (0, 1)$ and $v \in [0, 1]$. All these counterfactual parameters have a clear interpretation. For instance, $DMTR_d(y, v)$ gives the proportion of defendants with punishment resistance v who would have already recidivated after y periods since the court's final ruling if there were treated ($d = 1$) or not ($d = 0$). Analogously, $QMTR_d(\tau, v)$ and $AMTR_d(v)$ respectively provide the τ 's quantile and the average of the time-to-recidivate under treatment d , among defendants with punishment resistance v .

Based on these counterfactual objects, it is straightforward to define the Distributional, Quantile, and Average Marginal Treatment Effect functions:

$$DMTE(y, v) := DMTR_1(y, v) - DMTR_0(y, v), \quad (2.4)$$

$$QMTE(\tau, v) := QMTR_1(\tau, v) - QMTR_0(\tau, v), \quad (2.5)$$

$$MTE(v) := AMTR_1(v) - AMTR_0(v) = \int_0^1 QMTE(\tau, v) d\tau. \quad (2.6)$$

We also note that one can express $MTE(v)$ as a function of the $DMTE(y, v)$,¹⁰

$$MTE(v) = - \int_{\mathbb{R}_+} DMTE(y, v) dy.$$

Positive values of the QMTE and MTE functions indicate that punishment by fines and community services increases the defendant's time-to-recidivism compared to no punishment (so treatment is working as intended). On the other hand, positive values of the DMTE function indicate that punishment by fines and community services leads to an increase in the proportion of defendants who recidivate by time y compared to no punishment (so treatment is not working as intended). Putting it simply, for policy effectiveness in our context, positive values of QMTE

¹⁰This follows from the fact that, for any non-negative random variable I , $\mathbb{E}[I] = \int_{\mathbb{R}_+} (1 - \mathbb{P}[I \leq u]) du$.

and MTE are “good”, while negative values of DMTE are “good”.

Since we are dealing with a duration outcome subject to right-censoring, it is important to recognize early on that recovering $MTE(v)$ may be challenging, as it requires identifying the potential outcomes’ entire (conditional) counterfactual distribution. To somehow sidestep this limitation, it is common in the survival analysis literature to focus on a restricted version of the mean.¹¹ Following this rationale, we introduce a restricted version of the $MTE(v)$ function below, though we recognize that it is potentially less interesting than the $MTE(v)$.

Let γ_C denote the upper-bound of the support of the censoring variable C , that is, $\gamma_C := \inf \{c \in \overline{\mathbb{R}}: \mathbb{P}[C \leq c] = 1\}$. Define the restricted AMTR function as

$$RAMTR_d(v) := \mathbb{E}[\min\{Y^*(d), \gamma_C\} | V = v], \quad (2.7)$$

and the restricted average marginal treatment effect as

$$RMTE(v) := RAMTR_1(v) - RAMTR_0(v). \quad (2.8)$$

The RMTE is also connected with the DMTE function:

$$RMTE(v) = - \int_0^{\gamma_C} DMTE(y, v) dy,$$

which follows from the fact that, for a generic non-negative outcome W , $\mathbb{E}[\min\{W, \gamma_C\} | V = v] = \int_0^{\gamma_C} (1 - \mathbb{P}(W \leq y | V = v)) dy$. Of course, when $\gamma_C = \infty$ or if the support of $Y^*(d)$ is contained in the support of C (for the given v), $RMTE(v) = MTE(v)$.

Remark 1. Note that, when analyzing the impact of judicial decisions on recidivism, many papers¹² focus on distributional marginal treatment effects for small values of y (short-term analysis). In this paper, we advocate for moving beyond this short-term horizon by either considering a richer set of y ’s or by focusing on quantile or average marginal treatment effects of duration outcomes. In Appendix E.2, we numerically exemplify why focusing on duration outcomes may provide more information than the standard approach in Crime Economics.

3 Econometric Framework and Identification Results

We face some challenges in identifying the causal parameters of interest described in the previous section. As usual, potential outcomes are only (potentially) observed under one treatment status, i.e., $Y_i^* = Y_i^*(1) \cdot D_i + Y_i^*(0) \cdot (1 - D_i)$. Furthermore, our outcome of interest is subject to right-censoring, implying that we do not always observe Y^* but rather observe $Y_i = \min\{Y_i^*, C_i\}$, where C is the censoring variable. In the case of draws, we assume that Y_i^* happens before C_i , as is customary in survival analysis. Finally, we also expect that treatment

¹¹See, e.g., Karrison (1987), Zucker (1998), Chen and Tsiatis (2001), Andersen, Hansen and Klein (2004), Zhang and Schaubel (2012), among many others.

¹²See, e.g., Agan et al. (2023), Bhuller et al. (2019), Giles (2021), Huttunen et al. (2020), Klaassen (2021), and Possebom (2022).

statuses are related to the potential outcomes and potentially related to the censoring variable as well. Therefore, treatment is endogenous.

To tackle all these issues in a unified manner, we build on the MTE framework of Heckman and Vytlacil (2005) and extend it to handle duration outcomes.¹³ Towards this end, we consider a threshold-crossing treatment selection model

$$D = \mathbf{1}\{P(Z, C) \geq V\}, \quad (3.1)$$

where Z is an observed instrumental variable (with support $\mathcal{Z} \subset \mathbb{R}$), C is an observed censoring variable (with support $\mathcal{C} \subset \mathbb{R}_+$, and V is a latent heterogeneity that captures the unobserved treatment resistance (with support $(0, 1)$). The function $P : \mathcal{Z} \times \mathcal{C} \rightarrow \mathcal{P} \subseteq [0, 1]$ is unknown and captures the willingness to take the treatment for each value of Z and C . Importantly, our treatment selection model (3.1) imposes monotonicity (Imbens and Angrist, 1994; Vytlacil, 2002).

In order to better understand (3.1), let us go back to our empirical context and explain each component of it. Our instrumental variable Z is a measure of the trial judge’s leniency, which arguably does not affect time-to-recidivism other than through the judge’s decision to punish or not, especially when the judge’s allocation to the case is random, as in our application. Our censoring variable C captures the length of time between the defendant’s sentence date and the end of our sampling period, as is observed for all defendants. This variable can also capture seasonality patterns, as they are fully determined depending on the sentence’s date. The function P captures the trial judge’s punishment criteria, and it allows trial judges to update their punishment criteria over time (Bhuller and Sigstad, 2022) as it includes C as an argument. Finally, the variable V can be interpreted as unobserved punishment resistance, and it captures, among other things, the amount of criminal evidence in the defendant’s favor. The higher the V , the less likely the defendant will be punished, everything else equal. As already discussed before, Y^* captures the length of time between the defendant’s sentence date and her next criminal case’s starting date, and Y is the minimum of Y^* and time from the sentence’s date to the end of our sampling period, C .

3.1 Assumptions

In our setup, the available data for the researcher are $\{Y_i, C_i, D_i, Z_i\}_{i=1}^n$, while $Y_i^*(0)$, $Y_i^*(1)$, Y_i^* and V_i are latent variables. Henceforth, we assume that $\{Y_i, C_i, D_i, Z_i\}_{i=1}^n$ are independently and identically distributed as (Y, C, D, Z) . For simplicity, we drop exogenous covariates from the model and focus on the case with a single instrument. All results derived in the paper hold

¹³For duration models with and without endogeneity, see, e.g., Powell (1986); Chernozhukov and Hong (2002); Chernozhukov, Fernández-Val and Kowalski (2015); Frandsen (2015); Sant’Anna (2016); Delgado et al. (2022); Chen and Wang (2023); Fernández-Val, van Vuuren, Vella and Peracchi (2023). None of these papers build on the MTE framework as we do, or consider “irregular” parameters of interest.

conditionally on covariates and can be extended to the case with multiple instruments.¹⁴ Since we are dealing with a duration outcome variable, Y is non-negative by construction.

In what follows, we present a set of five assumptions (Assumptions 1-5) that will allow us to point-identify the DMTE and the QMTE functions across a range of thresholds and quantile points. They also allow one to identify the RMTE parameter. These assumptions are related to those imposed by Heckman and Vytlacil (2005) and Frandsen (2015) and involve assuming that censoring is not related to the potential outcomes $Y^*(d)$. Finally, we introduce additional support restrictions (Assumptions 6 and 7) that guarantee the identification of the entire DMTE and QMTE functions, implying that the MTE would also be identified.¹⁵ The last support assumption may be restrictive, but we include it for completeness.

Let us start with the first five assumptions and contextualize each of them to our empirical setup.

Assumption 1 (Random Assignment). *Conditional on C , the potential outcomes $Y^*(0)$, $Y^*(1)$ and V are independent of the instrument Z , i.e.,*

$$Z \perp\!\!\!\perp (Y^*(0), Y^*(1), V) | C.$$

Assumption 1 is an exogeneity assumption and is common in the literature about instrumental variables with censored outcomes (Frandsen, 2015). In our empirical application, this assumption holds conditional on the court district because trial judges are randomly assigned to cases within each court district.

Note also that Assumption 1 allows the instrument to depend on the censoring variable. In our empirical application, this flexibility is useful because the trial judge's punishment rate may depend on the case's sentence date if judges who entered the Judiciary more recently are more lenient than judges who retired at the beginning of our sampling period.

Assumption 2 (Propensity Score is Continuous). *Conditional on C , $P(z, c)$ is a nontrivial function of z and the random variable $P(Z, c) | C = c$ is absolutely continuous in Z , with support given by an interval $\mathcal{P} := [\underline{p}, \bar{p}] \subseteq [0, 1]$ for any $c \in \mathcal{C}$.¹⁶*

Assumption 2 is a rank condition, intuitively imposing that the instrument is locally relevant. In addition, we implicitly assume that the support of the propensity score does not vary with the value of C . In our application, this implicit assumption is plausible because the judges are mostly the same over time. Furthermore, the judge's lenience rate has a good amount of variation.

¹⁴As discussed by Mogstad, Torgovitsky and Walters (2021), the single threshold crossing model imposes strong homogeneity restrictions when we allow for multiple instruments.

¹⁵In Appendix G, we discuss three assumptions that allow censoring dependence and ensure partial identification.

¹⁶The assumption that \mathcal{P} is an interval is made for notational simplicity. All the proofs can be easily extended to the case where \mathcal{P} is a set with a non-empty interior.

Assumption 3 (V is continuous). *The distribution of the latent heterogeneity variable V conditional on C is absolutely continuous with respect to the Lebesgue measure.*

Assumption 3 is a regularity condition that allows us to normalize the marginal distribution of $V|C$ to be the standard uniform. Consequently, we can write $P(z, c) = \mathbb{P}[D = 1|Z = z, C = c]$ for any $z \in \mathcal{Z}$ and $c \in \mathcal{C}$. Moreover, this normalization implies that V is independent of C .

Assumption 4 (Overlap). *Conditional on C , all treatment groups exist, i.e., $\mathbb{P}[D = d|C = c] \in (0, 1)$ for any $d \in \{0, 1\}$ and any $c \in \mathcal{C}$.*

Assumption 4 is a regularity condition. It extends the standard overlap assumption in the policy evaluation literature to the setting with a duration outcome.

Assumption 5 (Random Censoring). *The censoring variables are independent of the uncensored potential outcomes given the latent heterogeneity V , i.e.,*

$$C \perp\!\!\!\perp (Y^*(0), Y^*(1)) | V.$$

Assumption 5 is an exogeneity assumption and is common in the literature about duration outcomes (Frandsen, 2015; Sant’Anna, 2016; Delgado et al., 2022). When combined with Assumption 3, Assumption 5 implies that C is unconditionally independent of the uncensored potential outcomes, i.e., $C \perp\!\!\!\perp (Y^*(0), Y^*(1))$. In our empirical application, this restriction imposes that the case’s sentence date is independent of the defendant’s decision to commit another crime in the future.

Importantly, Assumption 5 imposes that controlling for V accounts for all sources of endogeneity coming through the censoring variable. This assumption can be restrictive since endogeneity might still be present once controlling for the latent heterogeneity. If the researcher believes that this assumption is too strong in a particular application, she can use alternative assumptions that are sufficient to partially identify the distributional marginal treatment effect and some quantile marginal treatment effects when the outcome variable is right-censored. We discuss these alternative partial identification strategies in Appendix G.

Assumptions 1-5 are sufficient to identify the DMTE and QMTE functions across a range of thresholds y and quantiles τ . These are also sufficient to identify the RMTE parameter. However, to identify the marginal treatment effect (or the entire DMTE and QMTE functions), we need to impose the following additional restrictions.

Assumption 6 (Finite Moments). *Conditional on C , the potential outcome variables have finite first moments, i.e., $\mathbb{E}[|Y(d)||V = v, C = c] < \infty$ for any $d \in \{0, 1\}$, any $v \in [0, 1]$ and any $c \in \mathcal{C}$.*

Assumption 6 is a regularity condition that allows us to apply standard integration theorems and ensures that average treatment effects are well-defined.

Assumption 7 (Support Restriction). *The support of the uncensored potential outcomes is smaller than the support of the censoring variable, i.e., $\gamma_C = +\infty$ or $\gamma_d < \gamma_C$ for any $d \in \{0, 1\}$, where $\gamma_C := \inf \{c \in \overline{\mathbb{R}}: \mathbb{P}[C \leq c] = 1\}$ and $\gamma_d := \inf \{y \in \overline{\mathbb{R}}: \mathbb{P}[Y^*(d) \leq y] = 1\}$ for any $d \in \{0, 1\}$.*

Assumption 7 restricts the support of the potential outcomes of interest to be smaller than the censoring variable's support. In our empirical application, this assumption imposes that all defendants recidivate within ten years, which is the longest observation period in our sample. Formally, this restriction imposes that $\gamma_d < \gamma_C = 10$ years for any $d \in \{0, 1\}$. This rule out the possibility of defendants not recidivating until they die, and it is therefore not very plausible in our specific contest. We still present results using this assumption as they may be appropriate in empirical contexts different from ours.

3.2 Identification

We present our point-identification results that rely on Assumptions 1-5, 6 and 7.

First, define

$$\gamma_d(y, v, c) = \frac{d}{dv} \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = c].$$

We now state our main identification result: point-identification of the *DMTR* functions.

Proposition 3.1. *Suppose that Assumptions 1-5 hold. Then, for any $d \in \{0, 1\}$, $y < \gamma_C$ and $v \in \mathcal{P}$,*

$$DMTR_d(y, v) = (2d - 1) \cdot \mathbb{E}[\gamma_d(y, p, C) | P(Z, C) = v, C > y].$$

Proof. See Appendix A.2. ■

The above proposition shows how we can point-identify the distributional marginal treatment response for a given unobserved treatment resistance v . It involves first taking the derivative of the conditional joint distribution of the realized outcome Y and treatment status D given the propensity score $P = v$ and the censoring variable being above y ($C = y + \delta$ for $\delta > 0$) with respect to v , and then integrating over all values $\delta > 0$ such that the $y + \delta$ remains in the support of the censoring variable C . Contrary to the results in Carneiro and Lee (2009) and Carneiro et al. (2011), we need to tackle the right-censoring problem, which manifests in our results by having to condition on $C = y + \delta$, so that $C > y$, and then integrating over δ .

Furthermore, our results are specific to the $DMTR_d(y, v)$ function, and not for a generic transformation of $Y^*(d)$, say $G(Y^*(d))$ as in Carneiro and Lee (2009). This follows from the fact that we may not be able to identify the $DMTR_d(y, v)$ over all values of y in the support of

$Y^*(d)$, as a consequence of the censoring problem. Having said that, there are several functions that we can actually nonparametrically point-identify without additional restrictions and under standard regularity conditions, including the QMTE functions for a range of quantiles and the RMTE function. We state these results as a corollary for convenience. The proof is a direct consequence of the previous proposition, the definition of quantiles and the relationship of quantiles and expected values.

Corollary 3.1. *Suppose that Assumptions 1-5 and Assumption B.7 listed in the Appendix B hold. Then*

(a) *the QMTE(τ, v) function as defined in (2.5) is point-identified for any $v \in \mathcal{P}$ and $\tau \in (0, \bar{\tau}(v))$, where $\bar{\tau}(v) := \min \{\bar{\tau}_0(v), \bar{\tau}_1(v)\}$ and $\bar{\tau}_d(v) := DMTR_d(\gamma_C, v)$ for any $d \in \{0, 1\}$.*

(b) *the RMTE(v) function as defined in (2.8) is point-identified for any $v \in \mathcal{P}$.*

Notice that since the right-tail of the $DMTR_d(\cdot, v)$ may be differentially affected by the censoring problem, implying that $\bar{\tau}_1(v)$ may be different from $\bar{\tau}_0(v)$. As a consequence, we can only identify the QMTE(τ, v) over the common range of identified quantiles among treated and untreated units.

Another related remark worth stressing is that, under Assumptions 1-5, we cannot point-identify the MTE function (2.6). The rationale for this “negative” result is that we may never observe realizations of Y^* beyond γ_C when the support of C is smaller than the support of Y^* .¹⁷ In those cases, we cannot identify the right-tail of the distributional marginal treatment response, i.e., we cannot identify $DMTR_d(y, v)$ for $y \geq \gamma_C$. Of course, when the support of C is contained in the support of $Y^*(d)$, this situation does not arise, and we can identify the MTE function as long as it is well-defined. This is precisely what Assumptions 6 and 7 impose.

Corollary 3.2. *Suppose that Assumptions 1-7 and Assumption B.7 listed in the Appendix B hold. Then, MTE(v) is point-identified for any $v \in \mathcal{P}$.*

4 Estimation and Inference

In this section, we provide algorithms on how to semiparametrically estimate the DMTE, QMTE, MTE, and RMTE functions based on the identification results described in Proposition 3.1 and Corollaries 3.1 and 3.2. We discuss two sets of results in this section. First, we present generic algorithms to estimate the marginal treatment effect functionals that remain agnostic

¹⁷Recall that $\gamma_C := \inf \{c \in \mathbb{R}: \mathbb{P}[C \leq c] = 1\}$ denotes the least-upper bound of the censoring random variable C .

about the type of estimators used to estimate the nuisance functions. These results are useful to pin down the intuition and to provide templates for the type of flexible estimation procedures one can adopt. However, pinning down the asymptotic properties of such estimators at that level of generality is rather challenging, especially when it comes to inference. To ameliorate this, we provide estimation and inference procedures based on a more restricted class of estimators for the nuisance functions, though we formally establish their large sample properties.

4.1 Generic estimation procedure

We first present a generic algorithm one can use to estimate DMTE functions across a grid of threshold points $\{y_k\}_{k=0}^K$. The algorithm will make use of an estimator for the propensity score, $P(Z, C) = \mathbb{E}[D|Z, C]$, and the conditional distribution of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z, C)$ and C , $d \in \{0, 1\}$. This algorithm builds on Proposition 3.1. Recall that our data consist of *iid* observations $\{Y_i, C_i, D_i, Z_i\}_{i=1}^N$.

Algorithm 4.1 (Generic Estimation of DMTE function).

1. *Semiparametrically (or nonparametrically) estimate the propensity score $P: \mathcal{Z} \times \mathcal{C} \rightarrow [0, 1]$. Denote the fitted propensity score values by \hat{P}_i .*
2. *Define a grid of values for the duration outcome Y , $\{y_k\}_{k=0}^K$, such that $y_k > y_{k-1}$ for any $k \in \{1, \dots, K\}$ and $K \in \mathbb{N}$.*
3. *For each $k \in \{0, \dots, K\}$ and each $d \in \{0, 1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z, C)$, and C , that is,*

$$\Gamma(P, C; y_k, d) = \mathbb{E}[\mathbf{1}\{Y \leq y_k, D = d\} | P, C].$$

Since the propensity score for unit i , P_i , is unknown, use the estimated fitted values from Step 1. Denote the estimated fitted values by $\hat{\Gamma}(\hat{P}_i, C_i; y_k, d) = \hat{\Gamma}_{d,k,i}$.

4. *For each $k \in \{0, \dots, K\}$ and $d \in \{0, 1\}$, estimate the derivative of $\Gamma(P, C; y_k, d)$ with respect to P . Since $\Gamma(P, C; y_k, d)$ is unknown, use the estimated $\hat{\Gamma}_{d,k,i}$. Denote the estimated derivative evaluated at $P = v, C = c$, by $\hat{\gamma}_d(y_k, v, c)$, where $v \in \mathcal{P}$, and $c \in \mathcal{C}$.*
5. *For each $k \in \{0, \dots, K\}$ and each $d \in \{0, 1\}$, estimate $DMTR_d(y_k, v)$ by averaging $(2d - 1)\hat{\gamma}_d(y_k, v, c)$ over values of c such that $c \geq y_k$,*

$$\widehat{DMTR}_d(y_k, v) = (2d - 1)\hat{\mathbb{E}}\left[\hat{\gamma}_d(y_k, v, C) | C > y_k, \hat{P} = v\right],$$

where $\hat{\mathbb{E}}[\cdot]$ is a generic estimators for a conditional expectation.

6. *For each value $v \in \mathcal{P}$ and $d \in \{0, 1\}$, ensure that $\widehat{DMTR}_d(y_k, v)$ is non-decreasing in y_k , and bounded between zero and one.*

7. For each $k \in \{0, \dots, K\}$, estimate $DMTE(y_k, v)$ using

$$\widehat{DMTE}(y_k, v) := \widehat{DMTR}_1(y_k, v) - \widehat{DMTR}_0(y_k, v).$$

Building on Algorithm 4.1, it is straightforward to compute the QMTE functions. More specifically, from Step 6, we have that for both $d = 1$ and $d = 0$, $DMTR_d(y, v)$ is monotone in y for a given v , as any cumulative distribution function should be. Thus, one can invert these to compute the quantile marginal treatment response functions, and then take their differences to compute the QMTE function. More precisely, for each $d \in \{0, 1\}$ and a quantile τ , a generic estimator of the $QMTR_d(\tau, v)$ is given by

$$\widehat{QMTR}_d(\tau, v) := \min_{k \in \{0, \dots, K\}} \left\{ y_k : \widehat{DMTR}_d(y_k, v) \geq \tau \right\}.$$

The QMTE estimator for any $\tau \in [0, \bar{\tau}(v))$ is given by

$$\widehat{QMTR}(\tau, v) = \widehat{QMTR}_1(\tau, v) - \widehat{QMTR}_0(\tau, v).$$

One can also use get the RMTE function from the DMTE function by integrating the DMTE over y up to γ_C , that is,

$$\widehat{RMTE}(v) = - \int_0^{\gamma_C} \widehat{DMTE}(y, v) dy.$$

If the support of C is contained in the support of Y^* , as in Assumption 7, then $RMTE(v) = MTE(v)$.

Remark 2. The above procedures do not explicitly account for covariates X . However, every single step of Algorithm 4.1 can be thought as implicitly conditioning on covariate values $X = x$. In such cases, one would get a conditional version of the DMTE function, and, consequentially, a conditional version the QMTE, RMTE and MTE functions. If one were interested in the “unconditional” version of these functionals, all one needs to do is integrate the DMTE functional over values of the distribution of X given the propensity score $P = v$, and then follow the same steps as described in the paragraphs after Algorithm 4.1. If covariates are of moderate dimensions or data for each covariates strata is not-so-large to justify asymptotic approximations, one may be interested in imposing additional restrictions to bypass the “curse-of-dimensionality”. We follow this path in the next section.

4.2 Semiparametric Estimation and Inference Procedures

This section provides a more specific procedure to semiparametrically estimate the marginal treatment effect functionals discussed before. The steps we follow are similar to those in Algorithm 4.1, but we are more specific about the choice of nuisance functions. This degree of specificity allows us to establish large-sample statistical guarantees and provide asymptotically valid inference procedures for the target functionals of interest. Here, we also allow for additional covariates X to enter into the model, so we have data on $\{Y_i, C_i, D_i, Z_i, X_i\}_{i=1}^N$. In the

context of our application to the effect of alternative punishments to misdemeanor offenses on time-to-recidivism in Brazil, X is a set of court district indicators. For this reason, we focus on the case where all X 's are discrete.

We start discussing how we estimate the propensity score. Similar to Carneiro and Lee (2009), we model $P(Z, C, X) := \mathbb{E}[D|Z, C, X]$ using an additive partially linear series regression

$$P(Z, C, X) = \alpha_0 + X_i' \alpha_X + C_i \alpha_C + \varphi(Z), \quad (4.1)$$

where $(\alpha_0, \alpha_X', \alpha_C)'$ are unknown finite-dimensional parameters, and φ is an unknown (infinite-dimension) function. In our context, the partially linear additive specification (4.1) allows one to pool information from different court districts and run a single propensity score model for all courts. Alternatively, one could use a different propensity score model for each district, at the cost of getting arguably much less precise estimates.¹⁸

In practice, one would approximate $\varphi(\cdot)$ using a linear combination of the vector of basis functions $\psi^L(z) = (\psi_1(z), \psi_2(z), \dots, \psi_L(z))'$, for $L \in \mathbb{N}$. That is, $\varphi(z) \approx \psi^L(z)' \alpha_Z$, such that, as $L \rightarrow \infty$, the approximation error shrinks to zero. For simplicity, we pick a polynomial basis function, $\psi^L(z) = (z, z^2, \dots, z^L)'$, though other options such as B-splines are also possible; see, e.g., Chen (2007). Note that all the series coefficients can be estimated via ordinary least squares, i.e.,

$$\hat{\theta}^{fs} = \arg \min_{\theta^{fs} \in \Theta^{fs}} n^{-1} \sum_{i=1}^n (D_i - \alpha_0 - X_i' \alpha_X - C_i \alpha_C - \psi^L(Z_i)' \alpha_Z)^2 \quad (4.2)$$

where $\hat{\theta}^{fs} = (\hat{\alpha}_0, \hat{\alpha}_X', \hat{\alpha}_C, \hat{\alpha}_Z)'$. Thus, we can compute the fitted propensity score values

$$\tilde{P}_i = \tilde{P}(Z_i, C_i, X_i) = \hat{\alpha}_0 + X_i' \hat{\alpha}_X + C_i \hat{\alpha}_C + \psi^L(Z_i)' \hat{\alpha}_Z. \quad (4.3)$$

In finite samples, \tilde{P}_i might be negative or larger than one. To handle this, we follow Carneiro and Lee (2009) and use the trimmed version of \tilde{P}_i as our estimator,

$$\hat{P}_i = \tilde{P}_i + (1 - \epsilon - \tilde{P}_i) \mathbf{1}\{\tilde{P}_i > 1\} + (\epsilon - \tilde{P}_i) \mathbf{1}\{\tilde{P}_i < 0\}, \quad (4.4)$$

for a sufficiently small ϵ . In our application, we use $\epsilon = 0.01$. In our application, this is not material as only one single observation is trimmed.

Next, we move into the estimation of the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given P, C, X for $d \in \{0, 1\}$. Here, we follow the distribution regression approach introduced by Foresi and Peracchi (1995) and further formalized by Chernozhukov, Fernández-Val and Melly (2013). The idea is to pose a model with “varying coefficients” for the conditional distribution of $Y \cdot \mathbf{1}\{D = d\}$, $d \in \{0, 1\}$,

$$\Gamma(P, C, X; y, d) \equiv \mathbb{E}[\mathbf{1}\{Y \leq y, D = d\} | P, C, X]$$

¹⁸Alternatively, one could use semiparametric logit model $P(Z, C, X) = \exp\{\alpha_0 + X_i' \alpha_X + C_i \alpha_C + \varphi(Z)\} / (1 + \exp\{\alpha_0 + X_i' \alpha_X + C_i \alpha_C + \varphi(Z)\})$. In the Appendix, we show that our regularity conditions could still be satisfied. The same is true for a fully nonparametric series estimator.

$$= \Lambda(\beta_0(y, d) + X'\beta_X(y, d) + C\beta_C(y, d) + P\beta_P(y, d)) \text{ a.s.} \quad (4.5)$$

where $\theta_0(\cdot, \cdot) = (\beta_0(\cdot, \cdot), \beta_X(\cdot, \cdot)', \beta_C(\cdot, \cdot)', \beta_P(\cdot, \cdot)')' \mapsto \Theta \subseteq \mathbb{R}^{3+k_X}$ is a vector of nonparametric functions, k_X is the dimension of X , and Λ is a known link function.¹⁹ For concreteness, we focus on a logistic link function, $\Lambda(\cdot) = \exp(\cdot)/(1 + \exp(\cdot))$.

In order to estimate these unknown functions, we first need to acknowledge that the propensity score P_i is not observed. However, we can use the “generated regressor” \hat{P}_i in (4.4). Once we replace P_i with \hat{P}_i , we can then leverage the insights of Foresi and Peracchi (1995) and Chernozhukov et al. (2013) by noticing that, for a fixed y and d , (4.5) is a binary regression problem, allowing us to pointwise estimate these by maximizing the (feasible) conditional likelihood function

$$\hat{Q}(\theta; y, d) = \frac{1}{n} \sum_{i=1}^n \ln \ell_\theta(\mathbf{1}\{Y_i \leq y, D_i = d\}, X_i, C_i, \hat{P}_i; y, d) \quad (4.6)$$

with

$$\ell_\theta(b, x, c, p; y, d) = \Lambda(w'\theta)^b (1 - \Lambda(w'\theta))^{1-b},$$

and $w = (1, x', c, p)'$. Thus, the distribution regression (DR) estimator of $\theta_0(y, d)$ is given by

$$\hat{\theta}(y, d) = \arg \max_{\theta \in \Theta} \hat{Q}_n(\theta; y, d). \quad (4.7)$$

Notice that computing the DR estimators for several (y, d) points only requires running a sequence of binary regressions. This can be performed in any statistical software.

Next, note that the derivative of (4.5) with respect to P can be computed in closed-form for each (y, d) ,

$$\gamma_d(y, v, c, x) := \frac{d}{dv} \Gamma(v, c, x; y, d) = \beta_P(y, d) \Gamma(v, c, x; y, d) (1 - \Gamma(v, c, x; y, d)), \quad (4.8)$$

where we explored that Λ is the logistic link function. Denote the DR estimated fitted values of $\gamma_d(y, v, c, x)$ by

$$\hat{\gamma}_d(y, v, c, x) = \hat{\beta}_P(y, d) \hat{\Gamma}(v, c, x; y, d) (1 - \hat{\Gamma}(v, c, x; y, d)), \quad (4.9)$$

where the DR coefficients are as in (4.7), and

$$\hat{\Gamma}(v, c, x; y, d) = \frac{\exp(\hat{\beta}_0(y, d) + x'\hat{\beta}_X(y, d) + c\hat{\beta}_C(y, d) + v\hat{\beta}_P(y, d))}{1 + \exp(\hat{\beta}_0(y, d) + x'\hat{\beta}_X(y, d) + c\hat{\beta}_C(y, d) + v\hat{\beta}_P(y, d))}.$$

With (4.9) on hand, we can estimate

$$DMTR_d(y, v, x) := \mathbb{P}[Y^*(d) \leq y | V = v, X = x].$$

Let $n_{d,x,y} = \sum_{i=1}^n \mathbf{1}\{D_i = d, X_i = x, C_i > y\}$ denote the sample size with treatment status d , covariate value x , and censoring variable above y . Our proposed estimator for $DMTR_d(y, v, x)$

¹⁹This class of distribution regression models nests and extends many traditional duration models such as Cox (1972) proportional hazard model and Kalbfleisch and Prentice (1980) accelerated time model; see Delgado et al. (2022) for a discussion.

is given by

$$\widehat{DMTR}_d(y, v, x) = (2d - 1) \frac{\sum_{i=1}^n \mathbf{1}\{D_i = d, X_i = x, C_i > y\} \hat{\gamma}_d(y, v, C_i, x)}{n_{d,x,y}}. \quad (4.10)$$

Since $\widehat{DMTR}_d(y, v, x)$ is an estimator for a conditional distribution, it needs to be non-decreasing in y for all $(d, v, x) \in \{0, 1\} \times \mathcal{P} \times \mathcal{X}$. In finite samples, though, this may not be the case. We recommend using the rearrangement procedure of Chernozhukov, Fernandez-Val and Galichon (2009), also adopted by Wüthrich (2019).

Based on (4.10), we can then estimate the $DMTE(y, v, x) := DMTR_1(y, v, x) - DMTR_0(y, v, x)$ using

$$\widehat{DMTE}(y, v, x) := \widehat{DMTR}_1(y, v, x) - \widehat{DMTR}_0(y, v, x). \quad (4.11)$$

Analogously, one can estimate $QMTE(\tau, v, x)$ and $RMTE(v, x)$ functionals using

$$\widehat{QMTE}(\tau, v, x) = \widehat{QMTR}_1(\tau, v, x) - \widehat{QMTR}_0(\tau, v, x) \quad (4.12)$$

$$\widehat{RMTE}(v, x) = - \int_0^{\gamma_C} \widehat{DMTE}(y, v, x) dy, \quad (4.13)$$

respectively, where $\widehat{QMTR}_d(\tau, v, x) = \inf\{y \in \mathbb{R}_+ : \widehat{DMTR}_d(y, v, x) \geq \tau\}$.

We summarize all these estimation steps in the following algorithm.

Algorithm 4.2 (Semiparametric Estimation of MTE functionals).

1. *Semiparametrically estimate the propensity score using the series partially linear model (4.1). Denote its trimmed fitted propensity score values by \hat{P}_i as defined in (4.4).*
2. *Define a grid of values for the duration outcome Y , $\{y_k\}_{k=0}^K$, such that $y_k > y_{k-1}$ for any $k \in \{1, \dots, K\}$ and $K \in \mathbb{N}$.*
3. *For each $k \in \{0, \dots, K\}$ and each $d \in \{0, 1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z, C)$, C , and X using the distribution regression model (4.5), with estimated DR coefficients (4.7).*
4. *For each $k \in \{0, \dots, K\}$ and $d \in \{0, 1\}$, estimate the derivative of DR model with respect to P as in (4.8). Denote its estimated fitted value for a given x by $\hat{\gamma}_d(y_k, v, c, x)$ as in (4.9).*
5. *For each $k \in \{0, \dots, K\}$ and each $d \in \{0, 1\}$, compute $\widehat{DMTR}_d(y_k, v, x)$ as in (4.10).*
6. *For each value $v \in \mathcal{P}$, $d \in \{0, 1\}$, and for a given x , ensure that $\widehat{DMTR}_d(y_k, v, x)$ is non-decreasing in y_k , and bounded between zero and one.*
7. *For each $k \in \{0, \dots, K\}$, estimate the $DMTE(y_k, v, x)$ using (4.11).*

8. Estimate the $QMTE(\tau, v, x)$ using (4.12), with

$$\widehat{QMTR}_d(\tau, v, x) = \min\{y \in \{y_k\}_{k=0}^K : \widehat{DMTR}_d(y, v, x) \geq \tau\}, \quad d \in \{0, 1\}.$$

9. Estimate the $RMTE(v, x)$ using (4.13).

The next theorem establishes the large-sample properties of our proposed estimators. We defer all the regularity conditions to the appendix to streamline the presentation. Let $\bar{\tau}(v, x) := \min\{\bar{\tau}_0(v, x), \bar{\tau}_1(v, x)\}$ and $\bar{\tau}_d(v, x) := DMTR_d(\gamma_C, v, x)$ for any $d \in \{0, 1\}$.

Theorem 4.1. *Suppose that Assumptions 1-5, and Assumptions B.1-B.7 listed in Appendix B hold. Then,*

(a) for each fixed $y < \gamma_C$, $v \in \mathcal{P}$, and $x \in \mathcal{X}$,

$$\sqrt{n} \left(\widehat{DMTE}(y, v, x) - DMTE(y, v, x) \right) \xrightarrow{d} N(0, V_{y,v,x}^{dmte}),$$

with $\widehat{DMTE}(y, v, x)$ as defined in (4.11) and $V_{y,v,x}^{dmte}$ as defined in Appendix B.

(b) for each fixed $\tau \in (0, \bar{\tau}(v, x))$, $v \in \mathcal{P}$, and $x \in \mathcal{X}$,

$$\sqrt{n} \left(\widehat{QMTE}(\tau, v, x) - QMTE(\tau, v, x) \right) \xrightarrow{d} N(0, V_{\tau,v,x}^{qnte}),$$

with $\widehat{QMTE}(\tau, v, x)$ as defined in (4.12) and $V_{\tau,v,x}^{qnte}$ as defined in Appendix B.

(c) for each fixed $v \in \mathcal{P}$, and $x \in \mathcal{X}$,

$$\sqrt{n} \left(\widehat{RMTE}(v, x) - RMTE(v, x) \right) \xrightarrow{d} N(0, V_{v,x}^{rmte}),$$

with $\widehat{RMTE}(v, x)$ as defined in (4.13) and $V_{v,x}^{rmte}$ as defined in Appendix B.

Theorem 4.1 follows from first deriving the influence function of the $DMTR$ functions (4.10), paying particular attention to quantifying the estimation effect arising from replacing the true propensity score with the estimated one. After this step, all the results follow from the functional delta method and the continuous mapping theorem. The proof strategy is similar to Rothe (2009).

Although Theorem 4.1 indicates that one can potentially conduct inference using plug-estimates of the variance, in practice, that involves estimating additional nuisance functions and can be cumbersome. We propose using a weighted bootstrap procedure as in Ma and Kosorok (2005a) and Chen and Pouzo (2009) to avoid that. This bootstrap procedure is very straightforward to implement, as described in the next algorithm.

Algorithm 4.3 (Weighted-Bootstrap Implementation).

1. Estimate $DMTE$, $QMTE$, and $RMTE$ according to Algorithm 4.2.

2. Generate $\{V_i, i = 1, \dots, n\}$ as a sequence of independent and identically distributed non-negative random variables with mean one, variance one, and finite third moment (e.g., $V_i \sim \text{Exp}(1)$).
3. Compute the propensity score coefficients associated with (4.1) by minimizing the weighted least squares function, i.e.,

$$\hat{\theta}^{fs,*} = \arg \min_{\theta^{fs} \in \Theta^{fs}} n^{-1} \sum_{i=1}^n V_i \left(D_i - \alpha_0 - X_i' \alpha_X - C_i \alpha_C - \psi^L(Z_i)' \alpha_Z \right)^2 \quad (4.14)$$

where $\hat{\theta}^{fs,*} = (\hat{\alpha}_0^*, \hat{\alpha}_X^{*'}, \hat{\alpha}_C^*, \hat{\alpha}_Z^{*'})'$. Denote its trimmed fitted propensity score values by \hat{P}_i^* as defined in (4.4), but with $\hat{\theta}^{fs,*}$ in place of $\hat{\theta}^{fs}$.

4. Consider the same grid of values for the duration outcome Y as defined in Step 2 of Algorithm 4.2.
5. For each $k \in \{0, \dots, K\}$ and each $d \in \{0, 1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z, C)$, C , and X using the distribution regression model (4.5) with estimated DR coefficients

$$\hat{\theta}^*(y, d) = \arg \max_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^n V_i \ln \ell_{\theta}(\mathbf{1}\{Y_i \leq y, D_i = d\}, X_i, C_i, \hat{P}_i^*; y, d). \quad (4.15)$$

6. Follow Steps 4-9 of Algorithm 4.2 using $\hat{\theta}^*(y, d)$ instead of $\hat{\theta}(y, d)$. Denote by $\widehat{DMTE}^*(y_k, v, x)$, $\widehat{QMTE}^*(\tau, v, x)$, and $\widehat{RMTE}^*(v, x)$ the distributional, quantile, and restricted marginal treatment effects estimates.
7. Repeat Steps 2-6 B times, e.g., $B = 999$, and collect $\left\{ \left(\widehat{DMTE}^*(y_k, v, x) \right)_b, b = 1 \dots, B \right\}$. Do the same for the $\widehat{QMTE}^*(\tau, v, x)$ and $\widehat{RMTE}^*(v, x)$.
8. Obtain the $(1 - \alpha)$ quantile of $\left\{ \left| \left(\widehat{DMTE}^*(y_k, v, x) - \widehat{DMTE}(y_k, v, x) \right)_b \right|, b = 1 \dots, B \right\}$, $c^{dmte,*}(y_k, v, x; \alpha)$. Compute the analogous critical values based on $\widehat{QMTE}^*(\tau, v, x)$ and $\widehat{RMTE}^*(v, x)$.
9. Construct the $1 - \alpha$ (pointwise) confidence interval for $DMTE(y_k, v, x)$ as $\widehat{C}^{dmte}(y_k, v, x) = [\widehat{DMTE}(y_k, v, x) \pm c^{dmte,*}(y_k, v, x; \alpha)]$. Define $\widehat{C}^{qmte}(\tau, v, x; \alpha)$ and $\widehat{C}^{rmte}(v, x; \alpha)$ analogously.

The next theorem establishes that our weighted bootstrap procedure has asymptotically correct coverage.

Theorem 4.2. *Under the assumptions of Theorem 4.1, for any $0 < \alpha < 1$, and for each $v \in \mathcal{P}$, $x \in \mathcal{X}$, $y < \gamma_C$, and $\tau \in (0, \bar{\tau}(v, x))$, for $n \rightarrow \infty$,*

$$(a) \mathbb{P} \left(DMTE(y_k, v, x) \in \widehat{C}^{dmte}(y_k, v, x; \alpha) \right) \rightarrow 1 - \alpha,$$

$$(b) \mathbb{P} \left(QMTE(\tau, v, x) \in \widehat{C}^{qmte}(\tau, v, x; \alpha) \right) \rightarrow 1 - \alpha,$$

$$(c) \mathbb{P} \left(RMTE(v, x) \in \widehat{C}^{rmte}(v, x; \alpha) \right) \rightarrow 1 - \alpha.$$

Note that all functionals in Theorems 4.1 and 4.2 provide a covariate-specific treatment effect. In our application's context, we can get court-district-specific DMTE, QMTE, and RMTE estimates of the effect of fines and community service sentences on time-to-recidivism. An advantage of this approach is that one can better understand treatment effect heterogeneity across districts. However, with 193 court districts in the State of São Paulo, it may be desirable to further aggregate the MTE functionals as a way to summarize the obtained effects.

There are several potential aggregations one could entertain. In our specific context, we decided to aggregate the court-district-specific MTE functionals across court districts using the proportion of cases per court district as weights. Let $w_x = \mathbb{P}(X = x)$ denote the probability of a covariate X takes the value x , which, in our case, denotes the true proportion of cases assigned to a court district x . Let $\widehat{w}_x = n^{-1} \sum_{i=1}^n \mathbf{1}\{X_i = x\}$ be the plugin estimator of w_x .

For each $d \in \{0, 1\}$, $y \in \mathcal{Y}$ and $v \in \mathcal{P}$, let

$$DMTR_d^{avg}(y, v) = \mathbb{E} [DMTR_d(y, v, X)] = \sum_{x \in \mathcal{X}} w_x DMTR_d(y, v, x).$$

Analogously, let $DMTE^{avg}(y, v) = DMTR_1^{avg}(y, v) - DMTR_0^{avg}(y, v)$, $QMTE^{avg}(\tau, v) = QMTR_1^{avg}(\tau, v) - QMTR_0^{avg}(\tau, v)$, and $RMTE^{avg}(v) = - \int_0^{\gamma_C} DMTE^{avg}(y, v) dy$, where $QMTR_d^{avg}(\tau, v) := \inf\{y \in \mathbb{R}_+ : DMTR_d^{avg}(y, v) \geq \tau\}$. All these functionals can be straightforwardly estimated using functionals of

$$\widehat{DMTR}_d^{avg}(y, v) = \sum_{x \in \mathcal{X}} \widehat{w}_x \widehat{DMTR}_d(y, v, x),$$

with $\widehat{DMTR}_d(y, v, x)$ as in (4.10), just like in Equations (4.11)-(4.13). Their large-sample properties follow directly from the delta method and are summarized in the following corollary.

Corollary 4.1. *Suppose that Assumptions 1-5, and Assumptions B.1-B.7 listed in Appendix B hold. Then,*

(a) *for each fixed $y < \gamma_C$, and $v \in \mathcal{P}$,*

$$\sqrt{n} \left(\widehat{DMTE}^{avg}(y, v) - DMTE^{avg}(y, v) \right) \xrightarrow{d} N(0, V_{y,v}^{dmte,avg}).$$

(b) *for each fixed $\tau \in (0, \bar{\tau}(v, x))$, and $v \in \mathcal{P}$,*

$$\sqrt{n} \left(\widehat{QMTE}^{avg}(\tau, v) - QMTE^{avg}(\tau, v) \right) \xrightarrow{d} N(0, V_{\tau,v}^{qmte,avg}).$$

(c) *for each fixed $v \in \mathcal{P}$,*

$$\sqrt{n} \left(\widehat{RMTE}^{avg}(v) - RMTE^{avg}(v) \right) \xrightarrow{d} N(0, V_v^{rmte,avg}).$$

It is also straightforward to construct a weighted-bootstrap confidence interval for these functionals by using $\hat{w}_x^* = n^{-1} \sum_{i=1}^n V_i \mathbf{1}\{X_i = x\}$ as weights for the MTE functionals. We omit a detailed description to avoid repetition.

Remark 3. We note that other aggregations of the covariate-specific marginal treatment effect functionals do exist, but may be more challenging to estimate. For instance, one may be interested in functionals of the “unconditional” $DMTR_d(y, v)$, defined as

$$\begin{aligned} DMTR_d(y, v) &= \mathbb{E}[DMTR_d(y, P, X)|D = d, P = v] \\ &= \int DMTR_d(y, v, \bar{x}) f_{X|D, P}(\bar{x}|D = d, P = v) d\bar{x}. \end{aligned}$$

It should be clear that $DMTR_d(y, v)$ is different from $DMTR_d^{avg}(y, v)$, though the latter can be more easily estimated as it does not require estimation of conditional densities as the former does. If one were to focus on $DMTR_d(y, v)$, all the uncertainty in estimating it would come from the estimation of the conditional density of X . This follows from Theorem 4.1 establishing that $DMTR_d(y, v, x)$ is \sqrt{n} -consistent with discrete X ’s, while $f_{X|D, P}$ converges at slower rates. Developing a higher-order asymptotic analysis for estimators for $DMTR_d(y, v)$ would be interesting. We leave a detailed analysis of it for future research.

5 Empirical Application

In our empirical application, we answer the question: “Do alternative sentences such as fines and community service impact time-to-recidivism?”. In Subsection 5.1, we assess the plausibility of our identifying assumptions while, in Subsection 5.2, we describe the results of our empirical analysis using our proposed tools.

5.1 Assessing the Plausibility of our Assumptions

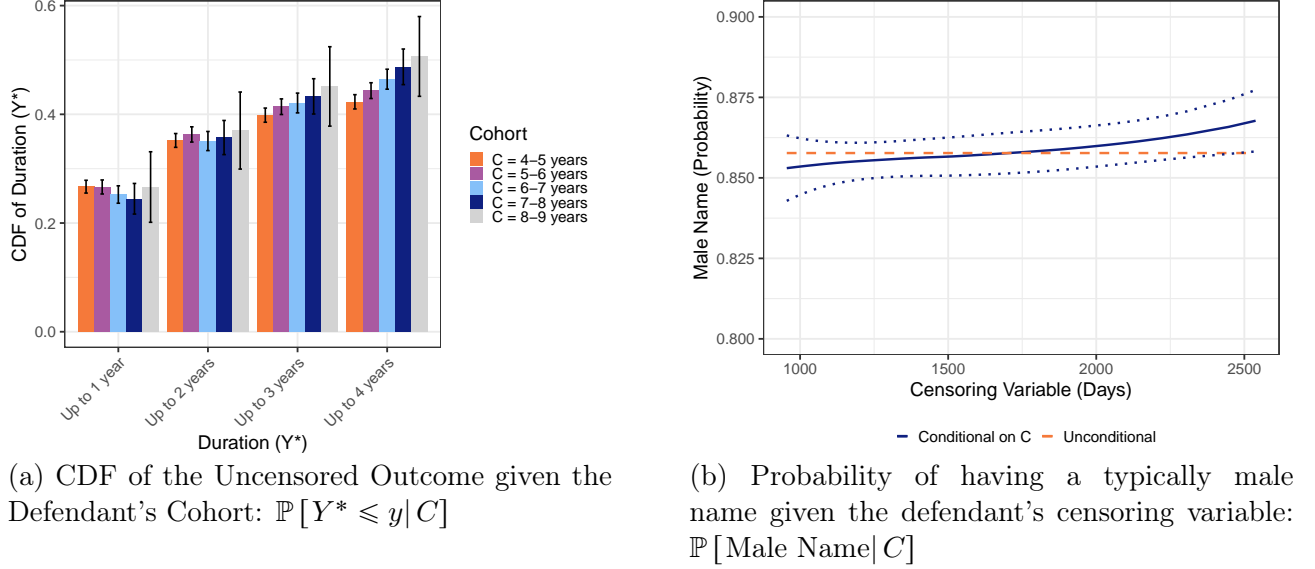
In this subsection, we use some descriptive statistics to assess the plausibility of two of our identifying assumptions: Random Censoring as in Assumption 5, and the support restriction introduced in Assumption 7.

Figure 3 focuses on assessing the plausibility of Assumption 5. Although it is not possible to directly test that the potential uncensored outcomes $(Y^*(0), Y^*(1))$ are independent of the censoring variable given the value of the latent heterogeneity V , it is possible to test whether the realized uncensored outcome (Y^*) is (approximately) independent of the censoring variable.

To do so, Sub-figure 3a shows the cumulative distribution function (CDF) of the uncensored outcome (Y^*) given cohorts based on the censoring variable. If Y^* is independent of the censoring variable, then this CDF should not vary across cohorts. Taking into account the sampling uncertainty, this figure shows that defendants’ cohorts are independent of the realized

outcome, indirectly suggesting that the censoring variable may be independent of the potential outcomes as imposed by Assumption 5.²⁰

Figure 3: Assessing the Plausibility of Assumption 5



Notes: Subfigure 3a shows the cumulative distribution function (CDF) of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable. Each color denotes a different cohort: orange denotes defendants who are observed for at least four years and at most five years during our sampling period, purple denotes defendants who are observed for at least five years and at most six years, light blue denotes defendants who are observed for at least six years and at most seven years, dark blue denotes defendants who are observed for at least seven years and at most eight years, and gray denotes defendants who are observed for at least eight years and at most nine years. These conditional CDFs are evaluated at four values of the uncensored potential outcome (one, two, three or four years), and these evaluation points are denoted in the x-axis. The y-axis denotes the value of the CDF, while black lines denote point-wise 99%-confidence intervals around the values of the CDF.

Subfigure 3b shows, as a solid dark blue line, the probability that the defendant has a typically male name as a function of their censoring variable. This nonparametric function was estimated using a local linear regression with an Epanechnikov kernel based on Calonico, Cattaneo and Farrell (2019). The bandwidth was optimally selected according to the IMSE criterion. The dotted dark blue lines are robust bias-corrected 99%-confidence intervals. The dashed orange line is the unconditional probability of having a typically male name.

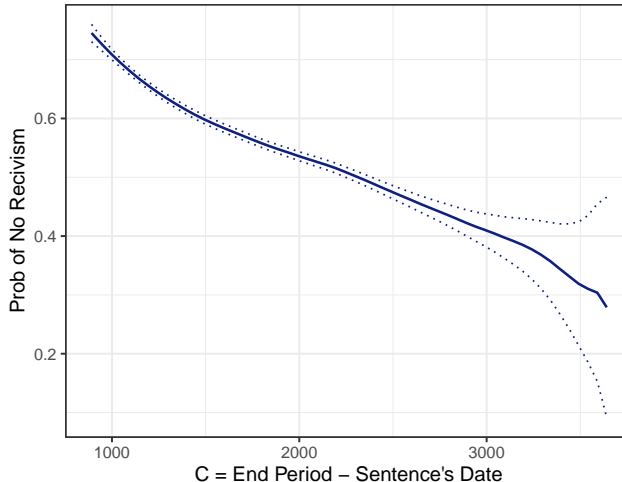
Another way to assess the plausibility of the random censoring assumption is to use a covariate-balancing test. In particular, we can analyze the relationship between the censoring variable and an excluded covariate — having a typically male name according to the Brazilian 2010 Census (*R* package `genderBR`). Figure 3b shows the probability of having a typically male name given the defendant's censoring variable (dark blue line). We find that, regardless of the censoring variable, this probability is close to the unconditional share of male names (orange line). Consequently, there is indirect and suggestive evidence that our random censoring restriction (Assumption 5) is valid.

Figure 4 shows the probability that a defendant does not recidivate during our sampling period given the value of her censoring variable. Conditioning on the defendants who stay the

²⁰More clearly, this figure suggests that the potential outcomes are negatively regression dependent on the censoring variable as imposed in Appendix G.1.

longest in our sample (large values of C), we still find a 30% probability that they do not recidivate during the observation period. This result suggests that our support restriction in Assumption 7 is not valid in this context. Although this result does not invalidate the analysis of the quantile marginal treatment effect, it implies that the restricted marginal treatment effect estimates should be interpreted carefully and should not be confused with estimates for the overall marginal treatment effect function.

Figure 4: Probability of No Recidivism during the Sampling Period: $\mathbb{P}[Y^* > C | C]$



Notes: Subfigure 4 shows the probability that a defendant does not recidivate during our sampling period given the value of her censoring variable. This nonparametric function was estimated using a local linear regression with an Epanechnikov kernel based on Calonico et al. (2019). The bandwidth was optimally selected according to the IMSE criterion. The dotted lines are robust bias-corrected 95% confidence intervals.

5.2 Empirical Results

In this section, we present our empirical results. Section 5.2.1 contains the information about the first stage of our estimation procedure, Equation (4.1). Section 5.2.2 presents our estimates of the target parameters in Corollary 4.1.

5.2.1 First Stage Results

We start by presenting the results of the first stage regression in our empirical analysis. In our model, the treatment variable D (“final ruling”) is a function of the instrument Z (“trial judge’s punishment rate”), the censoring variable C , and court district fixed effects. Following Subsection 4.2, we use a polynomial series to approximate the propensity score and report the estimated coefficients of a quadratic model in Table 2. Note that our instrument is strong according to the F-statistic of the first-stage regression. This result implies that Assumption 2 is plausible.

Table 2: First Stage Results

	Z	Z^2	C
Coefficient	0.663***	0.096	0.012***
Clusterized S.E.	(0.235)	(0.208)	(0.004)
F-statistic	817		

Note: The left-hand side variable is our treatment variable, i.e., D = “punished according to the final ruling in the case”. The standard errors are clusterized at the court district level. The third line reports the F-Statistic of a hypothesis test whose null is that the coefficients associated with Z and Z^2 are equal to zero. The first stage regression controls for court district fixed effects. To improve readability, we multiply the coefficient of the censoring variable (and its standard error) by 365. This transformation is equivalent to measuring the censoring variable in years instead of days.

We also report the distribution of the estimated propensity score in Figure 5. The blue histogram shows the distribution of the estimated propensity score given that defendant was punished (treated group) while the white histogram shows the distribution of the estimated propensity score given that defendant was not punished (control group). We find that most defendants have a probability of being punished around 50%. However, some defendants are more unlikely to be punished (estimated propensity score around 30%) and others are more likely to be punished (estimated propensity score around 70%). These widely spread propensity score distributions are positive for identification and estimation because they allow us to discuss DMTE, QMTE, and RMTE functions evaluated at many different points of the latent heterogeneity variable.

The vertical lines denote the unconditional 5th and 95th percentiles of the estimated propensity score. When discussing our results about the DMTE, QMTE and RMTE functions, we only report the estimates for latent heterogeneity values between these two percentiles. We do so to avoid extrapolation bias and to ensure the plausibility of Assumption 2.

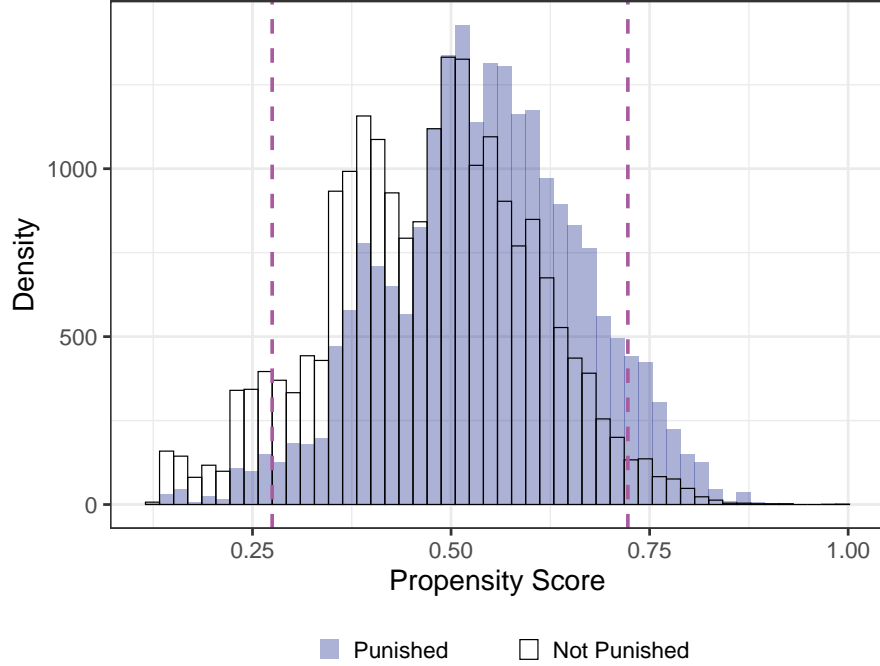
5.2.2 Estimated Target Parameters

To estimate the DMTE, QMTE and RMTE functions in our empirical application, we need to control for court district fixed effects. Consequently, we estimate 193 district-specific functions for each one of our treatment effect parameters (Theorem 4.1). To summarize our results, we average these functions over court districts using the proportion of cases per court district as weights, as in Corollary 4.1. We report the average DMTE function in Section 5.2.2.1, the average QMTE function in Section 5.2.2.2, and the average RMTE function in Section 5.2.2.3. Moreover, we compare our proposed methods against standard methods in the literature in Part 5.2.2.4.

5.2.2.1 Estimated DMTE function

Figure 6 shows the estimated average $DMTE(y, \cdot)$ functions for $y \in \{1, 2, \dots, 8\}$, where instead of measuring time-to-recidivism in days we measured it in years, which enhances read-

Figure 5: Distribution of the estimated propensity score given treatment status



Notes: The blue histogram shows the distribution of the estimated propensity score given that the defendant was punished (treated group). The white histogram shows the distribution of the estimated propensity score given that the defendant was not punished (control group). The vertical lines denote the unconditional 5th and 95th percentiles of the estimated propensity score.

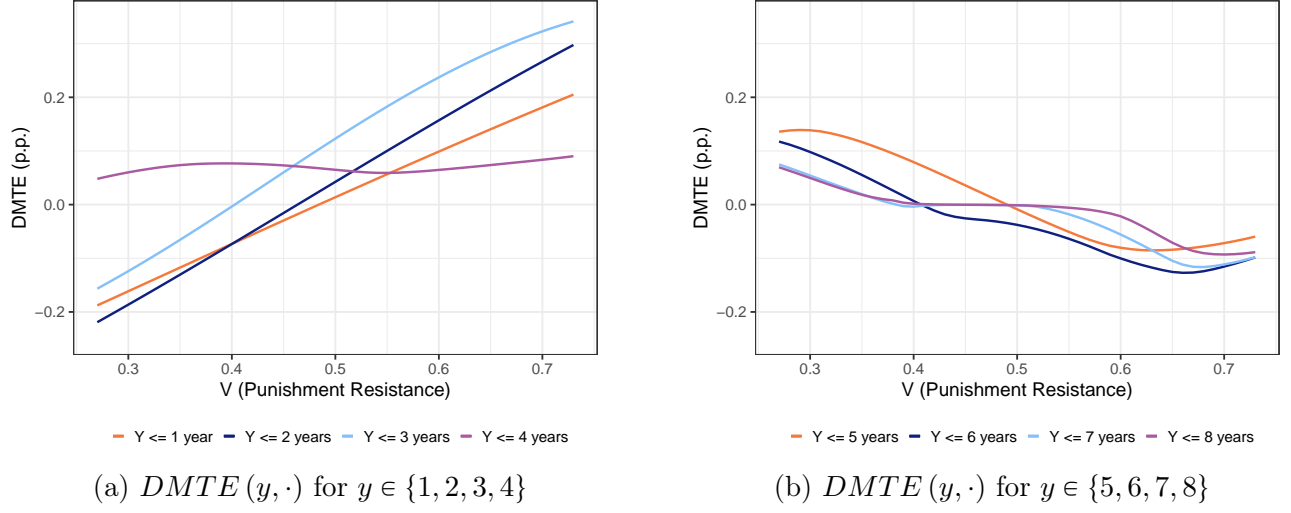
ability.²¹ These point estimates show relevant heterogeneity with respect to the treatment resistance (horizontal axis denotes values of V) and with respect to the recidivism horizon (different colors denote different values of y).

On the one hand, the $DMTE(y, \cdot)$ functions are increasing in the short-run ($y \in \{1, 2, 3\}$). This functional behavior indicates that defendants whom almost all judges would punish are less likely to recidivate, while defendants who would be punished only by tough judges are more likely to recidivate, at least in a given short-run time frame. This conclusion is supported by our 90%-confidence intervals (Figures D.2a-D.2c) since it is not possible to fit a horizontal line within them.

On the other hand, the $DMTE(y, \cdot)$ functions are increasing in the long-run ($y \in \{5, 6, 7, 8\}$), which is the sharp contrast to the “short-run” analysis. This functional behavior indicates that defendants whom almost all judges would punish would recidivate faster because of the punishment, while defendants who would be punished only by tough judges would take longer to recidivate as a result of being punished via alternative sentencing.

²¹In our data, we observe time-to-recidivism in days. To illustrate the readability improvements of writing the $DMTE$ function in years instead of days, we focus on one value of the time-to-recidivism variable. The $DMTE(y, \cdot)$ function when $y = 2$ shows the distributional marginal treatment effect given by $\mathbb{P}[Y^*(1) \leq 2 \cdot 365 \text{ days} \mid V = v] - \mathbb{P}[Y^*(0) \leq 2 \cdot 365 \text{ days} \mid V = v]$.

Figure 6: $DMTE(y, \cdot)$ for $y \in \{1, 2, \dots, 8\}$



Notes: Solid lines are the point estimates for the average $DMTE(y, \cdot)$ functions indicated in the legend of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported in Appendix D.1. These confidence intervals were computed using the weighted bootstrap, clustered at the court district level (Subsection 4.2).

This rich heterogeneity illustrates the importance of considering different time horizons and treatment resistance levels. If policymakers care more deeply about short-run recidivism, our point estimates suggest that designing sentencing guidelines that encourage strict judges to become more lenient could increase time-to-recidivism. However, if policymakers care more deeply about long-run recidivism, our point estimates suggest that designing sentencing guidelines that encourage lenient judges to become stricter could increase time-to-recidivism. Importantly, the results above highlight that if two researchers were to focus only on a specific but different time frame when defining whether a defendant recidivates or not may get very different answers, highlighting that the common practice of “binarizing” duration outcomes may come with important caveats.

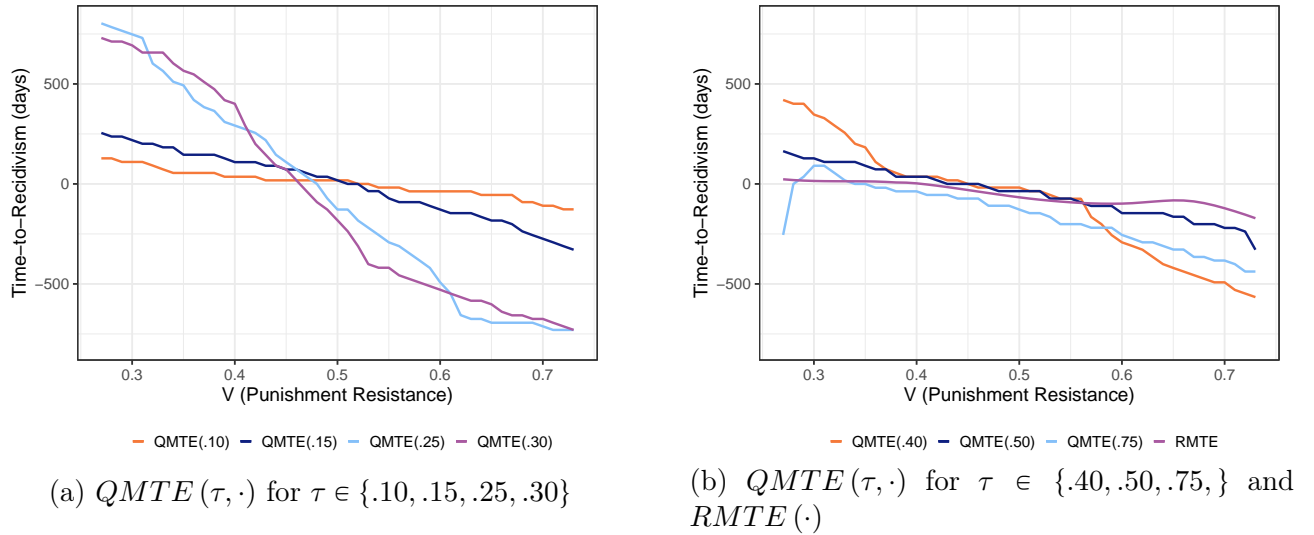
Given that the shape of the DMTE functions with respect to V varies a lot when one changes the threshold y , one may wonder if DMTEs are the most informative summary measures of heterogeneous treatment effects with respect to unobserved punishment resistance. Although DMTEs answer well-posed and policy-relevant questions, it may indeed be hard to convey the main takeaway of the application. In the end, did fines and community services help increase the time-to-recidivism or not? Based on Figure 6, one would need to answer “depends”, which is less than optimal. In what follows, we show that these limitations can be minimized by focusing on other functionals of interest such as QMTE and RMTE.

5.2.2.2 Estimated QMTE function

To have a deeper understanding of the time trade-offs associated with the effect of punishment on time-to-recidivism, we now focus on the average quantile marginal treatment effect functions. These objects are easier to interpret than the DMTE functions because they express the underlying treatment effects in the same units as the time-to-recidivism outcomes, i.e., days before the first recidivism event.

Figure 7 shows the estimated average $QMTE(\tau, \cdot)$ functions for $\tau \in \{.10, .15, .25, .30, .40, .50, .75\}$. Once more, these point estimates show relevant heterogeneity with respect to the punishment resistance (horizontal axis denotes values of V). However, the time horizon heterogeneity, now captured by the different quantiles, seems less relevant when compared with Figure 6.

Figure 7: $QMTE(\tau, \cdot)$ for $\tau \in \{.10, .15, .25, .30, .40, .50, .75\}$ and $RMTE(\cdot)$



Notes: Solid lines are the point estimates for the average $QMTE(\tau, \cdot)$ and $RMTE(\cdot)$ functions indicated in the legend of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported in Appendix D.1. These confidence intervals were computed using the weighted bootstrap clusterized at the court district level (Subsection 4.2).

Although the level of the estimated $QMTE(\tau, \cdot)$ functions depends on the quantile, all functions are decreasing in the unobserved resistance to punishment. These point estimates suggest that defendants whom almost all judges would punish would take longer to recidivate, while defendants who would be punished only by tough judges would recidivate faster compared to situations that they would not be punished. This result is statistically significant for $\tau \in \{.10, .15, .25, .30, .40, .50\}$ at the 10% significance level, see Figures D.4, D.5a and D.5b in the Appendix.

We reach a similar conclusion when we analyze the $QMTE(\cdot, v)$ as a function of the quantiles for specific values of unobserved resistance to treatment. Figure D.1 in the Appendix shows the

average $QMTE(\cdot, v)$ for $v \in \{.3, .4, .5, .6, .7\}$. We find that this function is always positive for small values of the unobserved resistance to punishment, while it is always negative for large values of v .

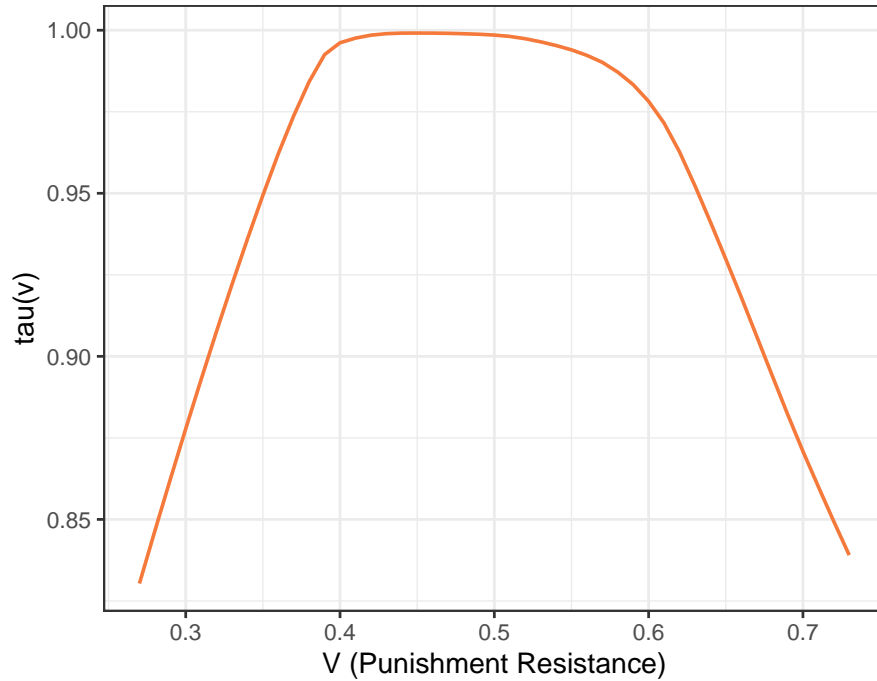
Overall, our QMTE point estimates suggest that designing sentencing guidelines that encourage strict judges to become more lenient could lead to increasing time-to-recidivism.

5.2.2.3 Estimated RMTE function

To have a single function that summarizes our results, we focus on the average $RMTE(\cdot)$. Figure 7b plots the point estimates of this function in purple. Before discussing this result, we must understand how restricted is the RMTE function (Equations (2.7) and (2.8)) compared to the overall MTE function. If the support of C is “too small” compared to the support of time-to-event outcome, RMTE may be further away from the MTE function, affecting its interpretability.

Figure 8 plots the estimated maximum identifiable quantile: $\bar{\tau}(v) := \min\{\bar{\tau}_0(v), \bar{\tau}_1(v)\}$ where $\bar{\tau}_d(v) := DMTR_d(\gamma_C, v)$ for any $d \in \{0, 1\}$ and $\gamma_C := \inf\{c \in \mathbb{R} : \mathbb{P}[C \leq c] = 1\}$. If the maximum identifiable quantile, $\bar{\tau}(\cdot)$ is “far away” from 1, then the support of C is “too small” compared to the support of time-to-event outcome.

Figure 8: Maximum Identifiable Quantile: $\bar{\tau}(v)$



Notes: The orange line plots the estimated maximum identifiable quantile, $\bar{\tau}(v)$, for each value of the unobserved resistance to treatment. The definition of $\bar{\tau}(v)$ can be found in Corollary 3.1.

Figure 8 shows that the RMTE function is almost an unrestricted mean for $v \in (.4, .5)$. However, the censoring problem is binding for small and large values of the unobserved resistance to treatment. Consequently, the RMTE is further away from the MTE function for extreme values of punishment resistance.

Now, analyzing the point estimates of the RMTE function (purple line in Figure 7b), we also find that the estimated restricted average marginal treatment effects decrease with the unobserved resistance to treatment. However, these point estimates are small in magnitude and statistically insignificant (Figure D.5d).

5.2.2.4 Comparison with Other Available Methods

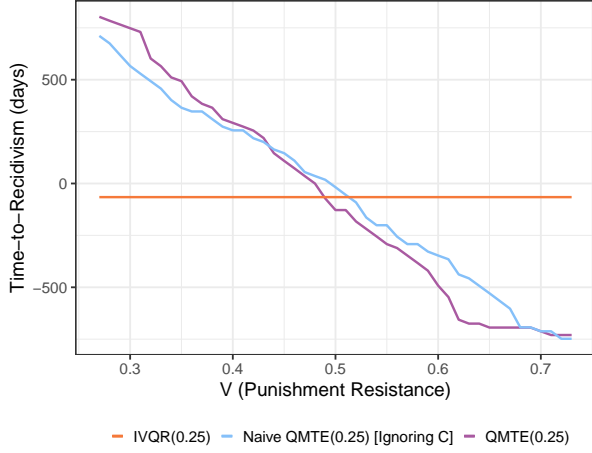
Here, we compare our proposed methods against other available methods in the literature. Differently from our approach, these estimates ignore that the outcome variable is right-censored and provide different conclusions when compared against our proposed estimator. For brevity, we focus our attention on the effects on the 25th and 50th percentiles ($QMTE(.25, \cdot)$ and $QMTE(.50, \cdot)$ functions) in Figure 9 and on the restricted average effect (RMTE function) in Figure 10.

Figure 9 focuses on the effect on the 25th and 50th percentiles of our outcome variable. Our proposed methods are illustrated by the purple lines. We have the average $QMTE(.25, \cdot)$ function in Figure 9a and the average $QMTE(.50, \cdot)$ function in Figure 9b (Corollary 4.1). The light blue lines denote a “naive” version of our estimators that follow the same steps as described in Section 4.2, but do not control for the censoring variable. The orange lines denote the standard method in the IV literature that takes into consideration endogenous selection into treatment but ignores (or aggregate) treatment effect heterogeneity with respect to unobserved resistance to treatment. The orange line in Figure 9a is the treatment coefficient of an IV quantile regression (Kaplan and Sun, 2017) for the 25th percentile, while Figure 9b is the treatment coefficient of an IV quantile regression (Kaplan and Sun, 2017) for the 50th percentile. Both IV quantile regressions use the censored outcome variable as the left-hand side variable, control for court district fixed effects and use the judge’s punishment rate as the instrument for the defendant being punished.

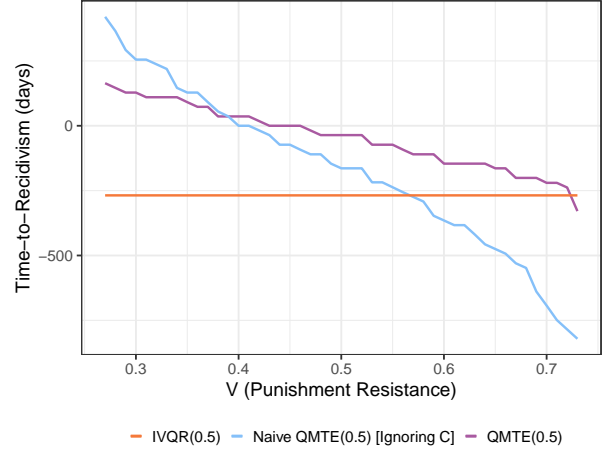
Analyzing Figures 9a and 9b, we find that the IV quantile regression does not capture the rich heterogeneity behind the treatment effects of fines and community service. In particular, the IV quantile regression estimates suggest a negative effect, ignoring that the treatment increases time-to-recidivism for some defendant types. Importantly, the IV quantile regression estimates do not lie entirely within the 90%-confidence intervals of the correctly estimated $QMTE(.25, \cdot)$ and $QMTE(.50, \cdot)$ functions (Figures D.4c and D.5b).

Moreover, in Figure 9a, we observe that our proposed estimator (purple line) and its naive

Figure 9: Comparing our Proposed Methods against Other Available Methods



(a) $QMTE(.25, \cdot)$ against Other Available Methods



(b) $QMTE(.50, \cdot)$ against Other Available Methods

Notes: Our proposed methods are illustrated by the purple lines. We have the average $QMTE(.25, \cdot)$ function in Figure 9a and the average average $QMTE(.50, \cdot)$ function in Figure 9b (Corollary 4.1). The light blue lines denotes a naive version of our estimators that ignores censoring. The orange lines are the treatment coefficients of IV quantile regressions (Kaplan and Sun, 2017) for the 25th and 50th percentiles.

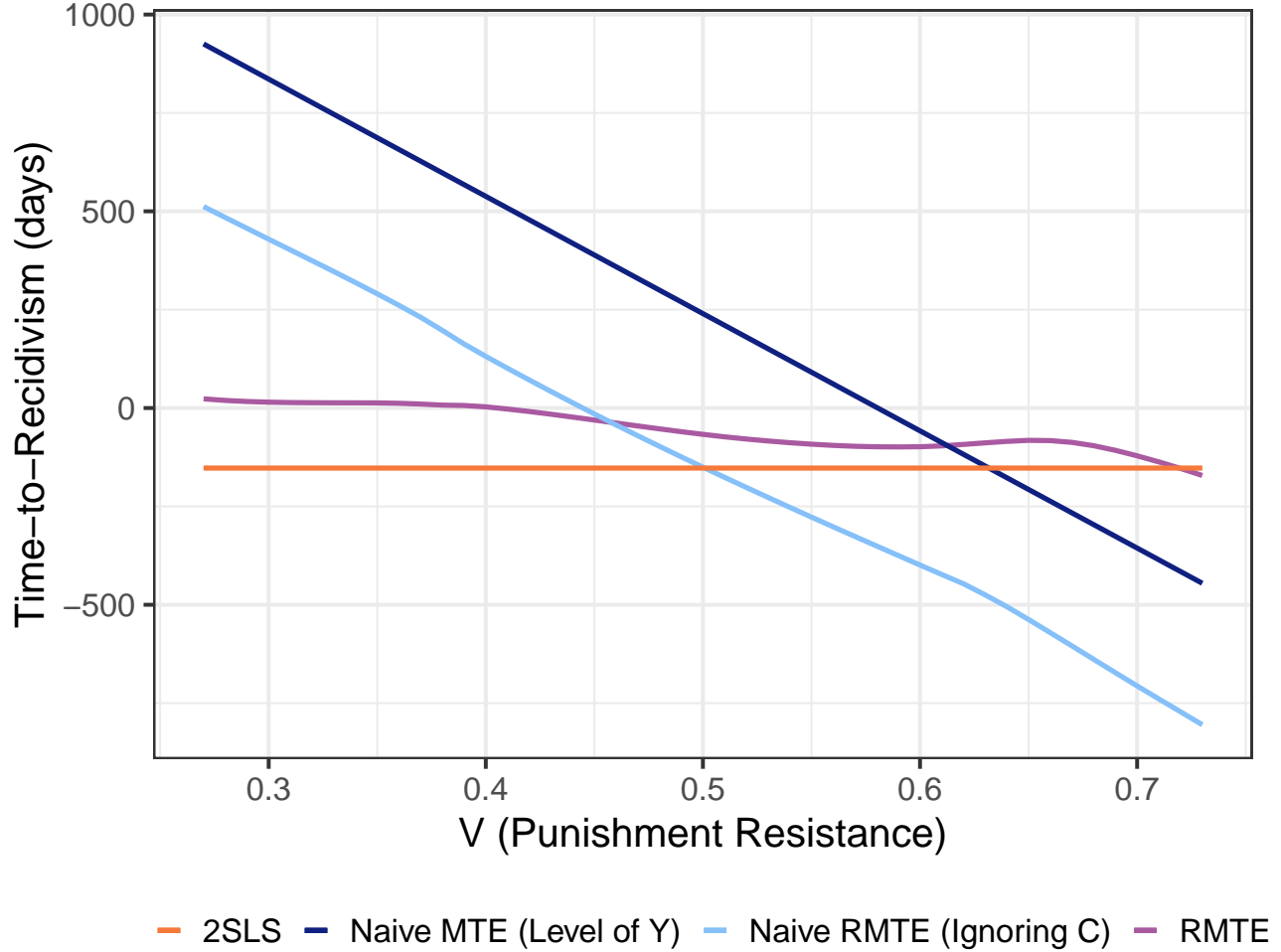
version (light blue line) reach similar point estimates. This finding is unsurprising because the estimated $QMTR_d(.25, \cdot)$ functions are always smaller than 2.5 years, and all defendants are observed for at least 2 years. Consequently, the censoring problem is not binding for low percentiles.

However, the censoring problem is binding for higher percentiles. In Figure 9b, we focus on the $QMTR_d(.50, \cdot)$ function and find that our proposed estimator (purple line) and its naive version (light blue line) differ in relevant ways. For example, the naive estimator finds a much more negative effect for individuals with a high punishment resistance. Importantly, the naive estimates do not lie entirely within the 90%-confidence intervals of the correctly estimated $QMTE(.50, \cdot)$ function (Figure D.5b).

Figure 10 focuses on the restricted average effect ($RMTE$ function). Our proposed method (Corollary 4.1) is illustrated by the purple line. The light blue line denotes a “naive” version of our estimator that follows the same steps as described in Section 4.2, but does not control for the censoring variable. The orange line is the treatment coefficient of a 2SLS regression that uses the censored outcome variable as the left-hand side variable, controls for court district fixed effects and uses the judge’s punishment rate as the instrument for the defendant being punished. The dark blue line is the estimated average MTE function based on a parametric estimator (Cornelissen, Dustmann, Raute and Schonberg, 2016, Appendix B.2) that imposes a linear MTE curve and ignores censoring concerns by directly using the level of the censored

outcome variable.

Figure 10: RMTE against Standard Methods



Notes: The purple line shows the average RMTE function (Corollary 4.1). The light blue line denotes a naive version of our estimators that ignores censoring. The orange line is the treatment coefficient of a 2SLS regression. The dark blue line is the estimated average MTE function based on a parametric estimator (Cornelissen et al., 2016, Appendix B.2) that directly uses the level of the censored outcome variable.

Analyzing Figure 10, we find two interesting results. First, the standard MTE method (dark blue line) and the naive version of our estimator (light blue line) exacerbate the heterogeneity that exists in the estimated *RMTE* function (purple line), finding much larger estimates in magnitude. Importantly, both blue lines do not lie entirely within the 90%-confidence intervals of the correctly estimated *RMTE* function (Figure D.5d).

Second, the 2SLS estimate finds a more negative effect of punishment on time-to-recidivism, suggesting that punishing defendants with fines and community service has led to faster recidivism than those obtained using our preferred method. Importantly, the 2SLS estimate does not lie entirely within the 90%-confidence intervals of the correctly estimated *RMTE* function

(Figure D.5d).

6 Conclusion

In this paper, we identify the distributional marginal treatment effect ($DMTE$), the quantile marginal treatment effect ($QMTE$) and the restricted marginal treatment effect ($RMTE$) functions when the outcome variable is right-censored. To do so, we extend the MTE framework (Heckman et al., 2006; Carneiro and Lee, 2009) to scenarios with duration outcomes. In this section, we discuss in which contexts our proposed methodology can be used and deepen our empirical discussion.

Our methodology can be applied to many empirical problems that face two simultaneous identification challenges: endogenous selection into treatment and right-censored data. In our empirical application, we focus on the effect of a fine on defendants' time-to-recidivism. In this case, judges observe more information than the econometrician when making their decisions and time-to-recidivism is a right-censored variable. In labor economics, the same identification challenges appear when analyzing the effect of receiving unemployment benefits on unemployment spells. Moreover, in the health sciences, when studying the effect of a drug on survival time, a researcher has to address both identification problems too.²²

Concerning its empirical contribution, our work is inserted in the literature about the effect of fines and community service sentences on future criminal behavior. Five recent papers in this field were written by Huttunen et al. (2020), Giles (2021), Klaassen (2021), Possebom (2022), Lieberman et al. (2023). All of them focus on binary variables indicating recidivism within a pre-specified time period. Huttunen et al. (2020) and Giles (2021) find that this type of punishment increases the probability of recidivism in Finland and Milwaukee (a city in the State of Wisconsin in the U.S.), respectively. Klaassen (2021) finds that alternative sentences decrease the probability of recidivism in North Carolina (a state in the U.S.). Possebom (2022) finds that this type of punishment has a small and statistically insignificant effect on the probability of recidivism in São Paulo, Brazil. Finally, Lieberman et al. (2023) analyze five American states and find that court fees have no impact on recidivism.

Differently from these five papers, our outcome variable is time-to-recidivism. Using a continuous outcome instead of binary indicators allows for a finer analysis of the heterogeneous effects of fines and community service sentences on future criminal behavior and may conciliate the conflicting results in the previous literature. For example, we find that this type of punishment increases time-to-recidivism for some individuals while decreasing it for other in-

²²The effect of unemployment benefits is discussed by Chetty (2008) and Delgado et al. (2022). Medical treatments are analyzed by Sullivan, Zwaag, El-Zeky, Ramanathan and Mirvis (1993), Spiegel (2002) and Trinquart, Jacot, Conner and Porcher (2016).

dividuals. If the first type of individual is more common in North Carolina than in Milwaukee and Finland, our focus on essential heterogeneity may shed light on these conflicting results.

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Supporting Information

(Online Appendix)

A Proofs of the main results

We start by stating an auxiliary lemma that will be used to derive our main identification results.

Lemma A.1. *If Assumptions 1-4 hold, then, for any $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$,*

$$\mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta] = \int_0^v \mathbb{P}[Y^*(1) \leq y | C = y + \delta, V = v] dv \quad (\text{A.1})$$

and

$$\mathbb{P}[Y \leq y, D = 0 | P(Z, C) = v, C = y + \delta] = \int_v^1 \mathbb{P}[Y^*(0) \leq y | C = y + \delta, V = v] dv, \quad (\text{A.2})$$

If Assumption 5 holds too, then, for any $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$,

$$\mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta] = \int_0^v \mathbb{P}[Y^*(1) \leq y | V = v] dv \quad (\text{A.3})$$

and

$$\mathbb{P}[Y \leq y, D = 0 | P(Z, C) = v, C = y + \delta] = \int_v^1 \mathbb{P}[Y^*(0) \leq y | V = v] dv. \quad (\text{A.4})$$

A.1 Proof of Lemma A.1

Fix $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$. To prove (A.1), note that

$$\begin{aligned} & \mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta] \\ &= \mathbb{E}[\mathbf{1}\{Y \leq y\} \mathbf{1}\{P(Z, C) \geq V\} | P(Z, C) = v, C = y + \delta] \\ & \quad \text{by (3.1)} \\ &= \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} \mathbf{1}\{v \geq V\} | P(Z, y + \delta) = v, C = y + \delta] \\ & \quad \text{because } \mathbf{1}\{Y(1) \leq y\} = \mathbf{1}\{Y^*(1) \leq y\} \text{ when } C > y \\ &= \int_0^1 \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} \mathbf{1}\{v \geq \tilde{v}\} | P(Z, y + \delta) = \tilde{v}, C = y + \delta, V = \tilde{v}] d\tilde{v} \\ & \quad \text{by the Law of Iterated Expectations and Assumption 3} \\ &= \int_0^1 \mathbf{1}\{v \geq \tilde{v}\} \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} | P(Z, y + \delta) = v, C = y + \delta, V = \tilde{v}] d\tilde{v} \\ &= \int_0^v \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} | P(Z, y + \delta) = v, C = y + \delta, V = \tilde{v}] d\tilde{v} \\ &= \int_0^v \mathbb{P}[Y^*(1) \leq y | C = y + \delta, V = \tilde{v}] d\tilde{v} \end{aligned}$$

by Assumption 1.

We can prove (A.2) analogously.

To prove (A.3), observe that

$$\begin{aligned}
& \mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta] \\
&= \int_0^v \mathbb{P}[Y^*(1) \leq y | C = y + \delta, V = \tilde{v}] d\tilde{v} \\
&= \int_0^v \mathbb{P}[Y^*(1) \leq y | V = \tilde{v}] d\tilde{v} \\
&\quad \text{by Assumption 5.}
\end{aligned}$$

We can prove (A.4) analogously.

A.2 Proof of Proposition 3.1

Fix $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

First, note that Equations (A.3) and (A.4) imply that

$$\frac{\partial \mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta]}{\partial v} = \mathbb{P}[Y^*(1) \leq y | V = v] \quad (\text{A.5})$$

and

$$\frac{\partial \mathbb{P}[Y \leq y, D = 0 | P(Z, C) = v, C = y + \delta]}{\partial z} = -\mathbb{P}[Y^*(0) \leq y | V = v] \quad (\text{A.6})$$

according to the Leibniz Integral Rule.

Combining Equations (2.1) and (A.5)-(A.6), we prove that

$$DMTR_d(y, v) = (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v}$$

for any $d \in \{0, 1\}$.

Since the last equation holds for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$DMTR_d(y, v) = (2d - 1) \cdot \int_y^{+\infty} \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = c]}{\partial v} \cdot f_{C|P(Z, C)=v, C \geq y}(c) dc$$

for any $d \in \{0, 1\}$.

B Regularity Conditions and Semiparametric Estimation

In this appendix we elaborate on standard regularity conditions for our proposed identification and estimation results to work.

B.1 Main Regularity Conditions

Although identification does not rely on any parametric assumption, some of them aid the estimation procedure. Covariates are easily incorporated when semiparametric assumptions are made and the curse of dimensionality is avoided. Additionally, semiparametric assumptions demand less data. In this appendix, we follow Rothe (2009) closely, but adapt his setting for the case where the link function is known instead of unknown. For the rest of the section, we assume an *i.i.d* sample. In this context, we introduce the following assumption:

Assumption B.1 (Semiparametric CDF). *Let $\mathbb{P}[Y \leq y, D = d | P, C, X] = \Lambda(\beta_0(d, y) + \beta_C(d, y)C + \beta_P(d, y)P + X'\beta_X(d, y))$, where $\Lambda(\cdot)$ is a known link function up to a finite dimensional vector (such as the logistic link), which is continuously differentiable in the index. Let $\Lambda'(\cdot)$ be the derivative of $\Lambda(\cdot)$, which is continuous.*

For the sake of exposition, let $W_{y,d} = 1\{Y \leq y, D = d\}$, $H = \{1, C, P, X\}$, $\hat{H} = \{1, C, \hat{P}, X\}$, $H_v = \{1, C, v, X\}$, $\beta_{d,y} := (\beta_0(d, y), \beta_C(d, y), \beta_P(d, y), \beta_X(d, y))$ for any y and $d \in \{0, 1\}$. Taking the derivative with respect to P for $\Lambda(\cdot)$ for both $W_{y,1}$ and $W_{y,0}$, we get the $DMTE(y, v)$ as

$$DMTR_1(y, v) - DMTR_0(y, v) = \Lambda'(\beta_{1,y}H_v)\beta_P(1, y) - \Lambda'(\beta_{0,y}H_v)\beta_P(0, y)$$

If P was known, it would be easy to estimate the DMTE as in the parametric part.

Since P is not known, we can estimate P in a semiparametric first stage, and obtain estimates for $\beta_{d,y}$ from the following maximum-likelihood procedure.²³ We focus on $d = 1$ for the sake of exposition and denote the semiparametric first-stage estimates by \hat{P} . Define

$$Ln(\beta_{1,y}, \hat{P}) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i W_{y,1,i} \log[\Lambda(\beta_{1,y}\hat{H}_i)] + (1 - W_{y,1,i}) \log[1 - \Lambda(\beta_{1,y}\hat{H}_i)] \quad (\text{B.1})$$

with solution $\hat{\beta}_{1,y}(\hat{P})$. If P was known, we could use the following unfeasible standard maximum likelihood procedure:

$$Ln(\beta_{1,y}, P) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i W_{y,1,i} \log[\Lambda(\beta_{1,y}H_i)] + (1 - W_{y,1,i}) \log[1 - \Lambda(\beta_{1,y}H_i)] \quad (\text{B.2})$$

with solution $\hat{\beta}_{1,y}(P)$.

To analyze our semiparametric estimator (B.1), we need to ensure that the unfeasible estimator in (B.2) is well-behaved. To do so, we impose the following assumption:

²³In the semiparametric first stage, we can estimate P using a standard series estimator.

Assumption B.2 (Unfeasible Likelihood). *The maximum likelihood estimator of (B.2), follows standard regularity conditions from Newey and McFadden (1994) for consistency and asymptotic normality.*

Assumption B.2 ensures that standard parametric inference could be performed if P was observed, implying that $\hat{\beta}_{1,y}(P) \xrightarrow{P} \beta_{1,y}$. Since Λ is the logistic link, the result is standard.

To ensure that our semiparametric estimator is consistent and derive its asymptotic distribution, we need to impose that our propensity score estimator converges sufficiently fast and satisfy some regularity conditions. To do so, we follow Rothe (2009) and impose the following assumption.

Assumption B.3 (First stage assumptions). *Let \hat{P} satisfy:*

1. $\hat{P}_i - P_i = \frac{1}{N} \sum_j w_n(Z_i, C_i, X_i, Z_j, C_j, X_j) \phi_j + r_{in}$ with $\max_i \|r_{in}\| = o_p(N^{-\frac{1}{2}})$ and $\max_i |\hat{P}_i - P_i| = o_p(N^{-\frac{1}{4}})$ where $\phi_j = \phi(D_j, Z_j, C_j, X_j)$ is an influence function with $\mathbb{E}[\phi_j | Z_j, C_j, X_j] = 0$ and $\mathbb{E}[\phi_j^2 | Z_j, C_j, X_j] \leq \infty$ and weights $w_n(Z_i, C_i, X_i, Z_j, C_j, X_j) = o(N)$.
2. *There exists a space \mathcal{P} such that $\mathbb{P}(\hat{P} \in \mathcal{P}) \rightarrow 1$ and the integral between 0 and infinity with respect to the radius of the log of the covering number with respect to the l_∞ norm of the class of functions \mathcal{P} is finite.*

Assumption B.3 is a high-level condition on the estimator. The first part states that the estimator admits a certain asymptotic expansion, whereas the second part requires the estimator to take values in some well-behaved function space with probability approaching 1.²⁴

B.2 Additional Regularity Conditions

Besides the previously mentioned conditions, which are the key components of the semi-parametric procedure, we do need to add additional regularity conditions that ensure that our procedures work.

Assumption B.4. *Assume that $\mathbb{P}[Y \leq y, D = d | P, C, X], \mathbb{P}[D = d | P, C, X]$ are twice continuously differentiable in P .*

Assumption B.5. *Assume that the support of Z is known and is a Cartesian product of compact connected intervals on which Z has a probability density function that is bounded away from zero.*

Assumption B.6. *Assume that $\psi_l(z)$, for $l \in L$ are r_ψ -times continuously differentiable on the support of Z for $r_\psi \geq 2$, where $\psi_l(z)$ is used to approximate $\varphi(z)$, an unknown function.*

Assumption B.7. *Assume that $Y^*(d)$ is continuous with respect to the Lebesgue measure.*

Assumption B.4 assures that we can apply the Leibniz Integral Rule to identify the DMTR functions. Assumptions B.5 and B.6 are standard in the series estimation literature. In particular, Assumption B.6 implies that the asymptotic bias (due to the series approximation by regression splines) converges to zero at a rate of L^{-r_ψ} as the number of approximation functions, L , diverges to infinity. Assumption B.7 is a regularity condition that ensures point identification for our quantile results.

²⁴A standard series estimator satisfies Assumption B.3.

B.3 Proof of Theorem 4.1: Consistency and Asymptotic Normality

To ensure our estimators of the *DMTE*, *QMTE*, *MTE* are consistent and asymptotically normal, we first need to ensure consistency of the feasible estimator of all the components of the DMTE. Then, we use functional approximation results to show asymptotic results for *DMTE*, *QMTE*, *MTE*.

B.3.1 Consistency of the $\beta_{d,y}$ estimators

We need to prove asymptotic equivalence between the solution of Equations (B.1) and (B.2). Then, by Assumption B.2, we get the consistency of the feasible semiparametric estimator. Note that

$$\begin{aligned} \sup_{\beta_{1,y}} |Ln(\beta_{1,y}, \hat{P}) - Ln(\beta_{1,y}, P)| \\ \leq \left[\inf_{\beta_{1,y}} \min_i \{ \Lambda(\beta_{1,y} \hat{H}_i), \Lambda(\beta_{1,y} H_i), 1 - \Lambda(\beta_{1,y} \hat{H}_i), 1 - \Lambda(\beta_{1,y} H_i) \} \left(\sup_{\beta_{1,y}} \max_i |\Lambda(\beta_{1,y} \hat{H}_i) - \Lambda(\beta_{1,y} H_i)| \right) \right] \\ \leq \left[O(1) \left(\sup_{\beta_{1,y}} \max_i |\Lambda(\beta_{1,y} \hat{H}_i) - \Lambda(\beta_{1,y} H_i)| \right) \right] \\ = o_p(1), \end{aligned}$$

where the first inequality can be derived using standard algebraic manipulations. Moreover, the second inequality holds because $\Lambda(\cdot) \in (0, 1)$. Furthermore, note that $\Lambda(\cdot)$ is continuous and $\max_i |\hat{H}_i - H_i|$ converges due to Assumption B.3, implying that $\max_i |\Lambda(\beta_{1,y} \hat{H}_i) - \Lambda(\beta_{1,y} H_i)|$ converges due to the continuous mapping theorem. Finally, since the supremum over $\beta_{1,y}$ in the third line is also continuous, we can apply the continuous mapping theorem again to prove the last equality.

Furthermore, $Ln(\beta_{1,y}, P)$ is a standard parametric likelihood, implying that it converges uniformly in $\beta_{1,y}$ to its expectation (Newey and McFadden, 1994, Lemma 2.4). Formally, we have that

$$\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, P) - L(\beta_{1,y})| = o_p(1)$$

where $L(\beta_{1,y}) = \mathbb{E} [Ln(\beta_{1,y})] = \mathbb{E} [W_{y,1,i} \log(\Lambda(\beta_{1,y} H)) + (1 - W_{y,1,i}) \log(1 - \Lambda(\beta_{1,y} H))]$ is a non-random function that is continuous in $\beta_{1,y}$. Taken together, it follows from the triangle inequality that

$$\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, \hat{P}) - L(\beta_{1,y})| = o_p(1)$$

implying that $\hat{\beta}_{1,y}(P)$ is consistent whenever $L(\beta_{1,y})$ attains a unique maximum at the true value of the parameter, which is the case by our identification results and Assumption B.1.

As a consequence, the consistency of our feasible semiparametric estimator follows from Theorem 2.1 by Newey and McFadden (1994) via Assumption B.2.

B.3.2 Asymptotic Distribution of the $\beta_{d,y}$ estimators

Now, we derive the asymptotic distribution of our semiparametric estimator in (B.1). Let $Ln(\beta_{1,y}, \hat{P}_i)_\beta$, $Ln(\beta_{1,y}, P_i)_\beta$, $L(\beta_{1,y}, P_i)_\beta$ be the derivative with respect to β of the individual's feasible log-likelihood, unfeasible log-likelihood and true log-likelihood respectively (the score). Define similarly the second-order derivative.

From a standard second-order Taylor expansion of the semiparametric log-likelihood around $\beta_{1,y}$, we have that

$$\sqrt{N}(\hat{\beta}_{1,y}(\hat{P}) - \beta_{1,y}) = \left[\frac{1}{N} \sum_i Ln(\bar{\beta}_{1,y}, \hat{P}_i)_{\beta,\beta} \right]^{-1} \sqrt{N} \frac{1}{N} \sum_i Ln(\beta_{1,y}, \hat{P}_i)_\beta, \quad (\text{B.3})$$

where $\bar{\beta}_{1,y}$ is between the estimated and true values. By the first part of Assumption B.3 and the consistency of $\hat{\beta}_{1,y}(\hat{P})$, we know that,

$$\left[\frac{1}{N} \sum_i Ln(\bar{\beta}_{1,y}, \hat{P}_i)_{\beta,\beta} \right]^{-1} \xrightarrow{p} \mathbb{E} [L(\beta_{1,y}, P_i)_{\beta,\beta}]^{-1} =: \Sigma.$$

Now, we focus on the last term in (B.3):

$$\begin{aligned} \sum_i Ln(\beta_{1,y}, \hat{P}_i)_\beta &= \sum_i W_{y,1,i} \frac{\partial \log[\Lambda(\beta_{1,y}, \hat{H}_i)]}{\partial \beta} + (1 - W_{y,1,i}) \frac{\partial \log[1 - \Lambda(\beta_{1,y}, \hat{H}_i)]}{\partial \beta} \\ &= \sum_i W_{y,1,i} \begin{bmatrix} \frac{\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} \\ \frac{\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} C_i \\ \frac{\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} \hat{P}_i \\ \frac{\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{\Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} X_i \end{bmatrix} \\ &\quad + (1 - W_{y,1,i}) \begin{bmatrix} \frac{-\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{1 - \Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} \\ \frac{-\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{1 - \Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} C_i \\ \frac{-\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{1 - \Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} \hat{P}_i \\ \frac{-\Lambda'(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)}{1 - \Lambda(\beta_0(1,y) + \beta_C(1,y)C_i + \beta_P(1,y)\hat{P}_i + \beta_X(1,y)X_i)} X_i \end{bmatrix} \end{aligned}$$

Considering the path $P_e = (1 - e)P + e[\hat{P} - P]$, we take the path-wise derivative of $\sum_i Ln(\beta_{1,y}, P_i)_\beta$ at direction $\hat{P} - P$ (the derivative of the submodel P_e evaluated at $e = 0$). This object is denoted by $\sum_i Ln(\beta_{1,y}, P_i)_{\beta, P_i}$ and is equal to

$$\sum_i Ln(\beta_{1,y}, P_i)_{\beta, P_i} = \sum_i W_{y,1,i} \begin{bmatrix} \frac{\Lambda''(\beta_{1,y}, H_i)\Lambda(\beta_{1,y}, H_i) - \Lambda'(\beta_{1,y}, H_i)^2}{\Lambda(\beta_{1,y}, H_i)^2} \beta_P(1, y) [\hat{P}_i - P_i] \\ \frac{\Lambda''(\beta_{1,y}, H_i)\Lambda(\beta_{1,y}, H_i) - \Lambda'(\beta_{1,y}, H_i)^2}{\Lambda(\beta_{1,y}, H_i)^2} C_i \beta_P(1, y) [\hat{P}_i - P_i] \\ \left[\frac{\Lambda''(\beta_{1,y}, H_i)\Lambda(\beta_{1,y}, H_i) - \Lambda'(\beta_{1,y}, H_i)^2}{\Lambda(\beta_{1,y}, H_i)^2} P_i \beta_P(1, y) + \frac{\Lambda'(\beta_{1,y}, H_i)}{\Lambda(\beta_{1,y}, H_i)} \right] [\hat{P}_i - P_i] \\ \frac{\Lambda''(\beta_{1,y}, H_i)\Lambda(\beta_{1,y}, H_i) - \Lambda'(\beta_{1,y}, H_i)^2}{\Lambda(\beta_{1,y}, H_i)^2} X_i \beta_P(1, y) [\hat{P}_i - P_i] \end{bmatrix} \quad (\text{B.4})$$

$$+ \sum_i (1 - W_{y,1,i}) \left[\begin{aligned} & \frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2} \beta_P(1,y) [\hat{P}_i - P_i] \\ & \frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2} C_i \beta_P(1,y) [\hat{P}_i - P_i] \\ & \left[\frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2} P_i \beta_P(1,y) + \frac{-\Lambda'(\beta_{1,y}H_i)}{1-\Lambda(\beta_{1,y}H_i)} \right] [\hat{P}_i - P_i] \\ & \frac{-\Lambda''(\beta_{1,y}H_i)[1-\Lambda(\beta_{1,y}H_i)]-\Lambda'(\beta_{1,y}H_i)^2}{[1-\Lambda(\beta_{1,y}H_i)]^2} X_i \beta_P(1,y) [\hat{P}_i - P_i] \end{aligned} \right]$$

We also define $\mathbb{E}[Ln(\beta_{1,y}, P_i)_{\beta, P_i}]$ analogously.

With these results in hand, we go back to (B.3) and expand around the deviations of the true first stage:

$$\sqrt{N}(\hat{\beta}_{1,y}(\hat{P}) - \beta_{1,y}) = \Sigma \cdot \sqrt{N} \left(\frac{1}{N} \sum_i Ln(\beta_{1,y}, P_i)_{\beta} + \frac{1}{N} \sum_i Ln(\beta_{1,y}, P_i)_{\beta, P_i} \right) + o_p(1) \quad (\text{B.5})$$

where $\sum_i \frac{1}{N} Ln(\beta_{1,y}, P_i)_{\beta}$ is the usual estimate of the score, which has mean 0. Thus, if we can show that the second term also has mean 0, the asymptotic normality of our semiparametric estimator follows by a standard multivariate *CLT* for the vector $[\frac{1}{N} \sum_i Ln(\beta_{1,y}, P_i)_{\beta}, \frac{1}{N} \sum_i Ln(\beta_{1,y}, P_i)_{\beta, P_i}]$.

Since all the components of (B.4) have a similar structure, we can focus on one of them and the results are symmetric for the rest.

Consider $\frac{1}{N} \sum_i W_{y,1,i} \frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2} \beta_P(1,y) [\hat{P}_i - P_i]$.

For notation simplicity, let $\frac{\Lambda''(\beta_{1,y}H_i)\Lambda(\beta_{1,y}H_i)-\Lambda'(\beta_{1,y}H_i)^2}{\Lambda(\beta_{1,y}H_i)^2} \beta_P(1,y) =: A(\beta_{1,y}H_i)$.

Note that

$$\begin{aligned} & \frac{1}{N} \sum_i W_{y,1,i} A(\beta_{1,y}H_i) [\hat{P}_i - P_i] \\ &= \frac{1}{N^2} \sum_i \sum_j w_n(Z_i, C_i, Z_j, C_j) W_{y,1,i} A(\beta_{1,y}H_i) \phi_j + o_p(N^{-\frac{1}{2}}) \\ &= \frac{1}{N} \sum_i \mathbb{E}[w_n(Z_i, C_i, Z, C)] \mathbb{E}[W_{y,1,i} A(\beta_{1,y}H_i) | H, Z, C] | Z_i, C_i] \phi_i + o_p(N^{-\frac{1}{2}}) \end{aligned}$$

where the first equality is due to Assumption B.3 and the second equality is due to the *U*-statistics Hajek projection.

Now, by a standard law of large numbers, we have that

$$\begin{aligned} & \frac{1}{N} \sum_i \mathbb{E}[w_n(Z_i, C_i, Z, C)] \mathbb{E}[W_{y,1,i} A(\beta_{1,y}H_i) | H, Z, C] | Z_i, C_i] \phi_i + o_p(N^{-\frac{1}{2}}) \\ & \xrightarrow{p} \mathbb{E}[\mathbb{E}[w_n(Z_i, C_i, Z, C)] \mathbb{E}[W_{y,1,i} A(\beta_{1,y}H_i) | H, Z, C] | Z_i, C_i] \phi_i] \\ &= \mathbb{E}[\mathbb{E}[\mathbb{E}[w_n(Z_i, C_i, Z, C)] \mathbb{E}[W_{y,1,i} A(\beta_{1,y}H_i) | H, Z, C] | Z_i, C_i] \phi_i | Z_i, C_i]] \\ &= \mathbb{E}[\mathbb{E}[w_n(Z_i, C_i, Z, C)] \mathbb{E}[W_{y,1,i} A(\beta_{1,y}H_i) | H, Z, C] | Z_i, C_i] \mathbb{E}[\phi_i | Z_i, C_i]] \\ &= 0 \end{aligned}$$

where the last equality is due to Assumption B.3. Thus, a standard *CLT* assures the asymptotic normality of the estimator for the parametric part.

B.3.3 Asymptotic behaviour of the DMTE estimator

We derive the influence function of our estimator to be able to express the variance and to apply functional limit theory results. Since DMTE is the difference of two *DMTR* functions, we can focus on the influence function of one of the *DMTR* and then apply linearity to get the influence function of the estimator for the DMTE. We will express the *DMTR* as an unconditional moment that depends on two parameters $(\beta_{d,y}, P)$ that are themselves expressed as unconditional moments.

In particular, P is such that $\mathbb{E}(D|Z, C, X) = P$. Omitting X for simplicity, we can express the moment that determines P with some loss of efficiency as:

$$\mathbb{E}[CZ(D - P)] = 0 \quad (\text{B.6})$$

Similarly, recall that $L(\beta_{1,y}) = \mathbb{E}[Ln(\beta_{1,y})] = \mathbb{E}[W_{y,1,i} \log(\Lambda(\beta_{1,y}H)) + (1 - W_{y,1,i}) \log(1 - \Lambda(\beta_{1,y}H))]$ is a non-random function that is continuous and differentiable in $\beta_{1,y}$. Note $L(\beta_{1,y})$ is itself a function of P since P is inside H . We can rewrite this as a moment equality using the score as:

$$\mathbb{E}[S(\beta_{1,y}[P])] = \mathbb{E}\left[W_{y,1,i} \frac{\partial \log(\Lambda(\beta_{1,y}H))}{\partial \beta_{1,y}} + (1 - W_{y,1,i}) \frac{\partial \log(1 - \Lambda(\beta_{1,y}H))}{\partial \beta_{1,y}}\right] = 0 \quad (\text{B.7})$$

Let $N = (P, \mathbf{1}\{C > y\})$. Then, the moment that identifies the $DMTR_1$ is: $DMTR_1 = \mathbb{E}[\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)\hat{P})\beta_P(1, y) | N]$, or, using an unconditional representation with some efficiency loss,

$$\mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1) \cdot P \cdot \mathbf{1}\{C > y\}] = 0 \quad (\text{B.8})$$

To be more specific, the relevant moments that define the parameters of interest are

$$0 = \mathbb{E}[CZ(D - P)] \quad (\text{B.9})$$

$$0 = \mathbb{E}\left[W_{y,1} \frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}{\Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))} - (1 - W_{y,1}) \frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}{1 - \Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}\right] \quad (\text{B.10})$$

$$0 = \mathbb{E}\left[W_{y,1} \frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}{\Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))} C - (1 - W_{y,1}) \frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}{1 - \Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))} C\right] \quad (\text{B.11})$$

$$0 = \mathbb{E}\left[W_{y,1} \frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}{\Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))} P - (1 - W_{y,1}) \frac{\Lambda'((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))}{1 - \Lambda((\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P))} P\right] \quad (\text{B.12})$$

$$0 = \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1) \cdot P \cdot \mathbf{1}\{C > y\}] \quad (\text{B.13})$$

We follow Newey (1994), Ichimura and Newey (2022), Akerberg, Chen, Hahn and Liao (2014) and Hahn, Liao, Ridder and Shi (2021). We will assume that standard conditions for the interchange of integration and differentiation hold (such as dominated convergence theorem conditions).

Before proceeding, note that:

$$\mathbb{E}[CZ(D - P)] = \int_{C \times Z \times D} cz(d - P)f(c, z, d)d\mu(c, z, d) = \int_{\mathcal{D}} cz(d - P)f(\delta)d\mu(\delta) \quad (\text{B.14})$$

This also holds for the rest of the moments, where δ is the full data vector. This is useful to be able to replicate the form of Equation (3.10) in Newey (1994), which is instrumental in deriving the influence functions.

We want to derive the influence function of $DMTR_1$. Thus, we can start with (B.13) and consider a parametric submodel t for the nuisance parameters and differentiate. Let $s(\delta)$ be the score of the data.

$$\begin{aligned} & \frac{\partial \mathbb{E}_t[(\Lambda'(\beta_0(1, y)(t) + \beta_C(1, y)(t)C + \beta_P(1, y)P(t))\beta_P(1, y)(t) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \\ &= \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1)P\mathbf{1}\{C > y\}s(\delta)] \\ &+ \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y)(t) + \beta_C(1, y)(t)C + \beta_P(1, y)(t)P)\beta_P(1, y)(t) - DMTR_1)P\mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \\ &+ \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \end{aligned}$$

By the implicit function theorem, we have that

$$\begin{aligned} & \frac{\partial DMTR_1}{\partial t} \Big|_{t=0} \\ &= [-\mathbb{E}[P\mathbf{1}\{C > y\}]]^{-1} \left[\mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1)P\mathbf{1}\{C > y\}s(\delta)] \right. \\ &+ \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y)(t) + \beta_C(1, y)(t)C + \beta_P(1, y)(t)P)\beta_P(1, y)(t) - DMTR_1)P\mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \\ &+ \left. \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \right]. \end{aligned}$$

Now, we need to express the second two components as products with the score of the data to apply Equation (3.10) of Newey (1994). We start with

$$\frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y)(t) + \beta_C(1, y)(t)C + \beta_P(1, y)(t)P)\beta_P(1, y)(t) - DMTR_1)P\mathbf{1}\{C > y\}]}{\partial t},$$

which is the parametric part of the model. We have that

$$\begin{aligned} & \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y)(t) + \beta_C(1, y)(t)C + \beta_P(1, y)(t)P)\beta_P(1, y)(t) - DMTR_1)P\mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \\ &= \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P) \cdot \beta_P(1, y) \cdot P \cdot \mathbf{1}\{C > y\}] \end{aligned}$$

$$\cdot \left(\frac{\partial \beta_0(1, y)(t)}{\partial t} \Big|_{t=0} + \frac{\partial \beta_C(1, y)(t)}{\partial t} \Big|_{t=0} \cdot C + \frac{\partial \beta_P(1, y)(t)}{\partial t} \Big|_{t=0} \cdot P \right) \Big]$$

We claim, and will become evident further on that is indeed true, that, using Equations (B.9)-(B.12), we can express $\frac{\partial \beta_A(1, y)(t)}{\partial t} = \mathbb{E}[IF_{\beta_A(1, y)} s(\delta)]$, where $IF_{\beta_A(1, y)}$ is the influence function of $\beta_A(1, y)$ and A is a random variable.

Then, we have that

$$\begin{aligned} & \frac{\partial DMTR_1}{\partial t} \Big|_{t=0} \\ &= [-\mathbb{E}[P \mathbf{1}\{C > y\}]]^{-1} \\ & \cdot \left[E[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1)P \mathbf{1}\{C > y\} s(\delta)] \right. \\ & \quad + E[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y)P \mathbf{1}\{C > y\} \\ & \quad \cdot (\mathbb{E}[IF_{\beta_0(1, y)} s(\delta)] + \mathbb{E}[IF_{\beta_C(1, y)} s(\delta)]C + \mathbb{E}[IF_{\beta_P(1, y)} s(\delta)]P)] \\ & \quad \left. + \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t) \mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \right], \end{aligned}$$

or, equivalently,

$$\begin{aligned} & \frac{\partial DMTR_1}{\partial t} \Big|_{t=0} \\ &= [-\mathbb{E}[P \mathbf{1}\{C > y\}]]^{-1} \left[E[\{(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1)P \mathbf{1}\{C > y\} \right. \\ & \quad + \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y))P \mathbf{1}\{C > y\}]IF_{\beta_0(1, y)} \\ & \quad + \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y))P \mathbf{1}\{C > y\}]C IF_{\beta_C(1, y)} \\ & \quad \left. + \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y))P \mathbf{1}\{C > y\}]P IF_{\beta_P(1, y)}\} s(\delta)] \right. \\ & \quad \left. + \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t) \mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0} \right]. \end{aligned}$$

Now, we need to derive the non-parametric component

$$\frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t) \mathbf{1}\{C > y\}]}{\partial t} \Big|_{t=0}.$$

We assume that the conditions for the Riesz Representation Theorem hold for

$$\frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t) \mathbf{1}\{C > y\}]}{\partial t}.$$

See, for example, Akerberg et al. (2014) for such conditions as being a linear bounded functional. Then, there is a unique b in a properly defined space with the inner product $\langle b_1, b_2 \rangle = \mathbb{E}[b_1 b_2]$ such that:

$$\begin{aligned} & \frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t) \mathbf{1}\{C > y\}]}{\partial t} \\ &= \mathbb{E} \left[b \frac{\partial \{CZ(D - P(t))\}}{\partial t} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{\partial \mathbb{E}[bCZ(D - P(t))]}{\partial t} \\
&= \mathbb{E} \left[\tilde{b}(\delta) \frac{\partial P(t)}{\partial t} \right] \\
&= \frac{\partial \mathbb{E}[\tilde{b}(\delta)P(t)]}{\partial t}
\end{aligned} \tag{B.15}$$

where $\tilde{b}(\delta) = -bCZ$.

To find the IF following Ichimura and Newey (2022), we need to find a $\phi(\delta, P, \alpha)$ such that:

$$\begin{aligned}
&\frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t} \\
&= \int \phi(\delta, P, \alpha)G(d\delta)
\end{aligned} \tag{B.16}$$

where G is a perturbation from the true CDF .

Furthermore, from (B.9), we can see that

$$0 = \frac{\partial \mathbb{E}_t[CZ(D - P(t))]}{\partial t} = \int CZ(D - P)G(d\delta) + \frac{\partial \mathbb{E}[CZ(D - P(t))]}{\partial t}$$

or, equivalently,

$$\int CZ(D - P)G(d\delta) = -\frac{\partial \mathbb{E}[CZ(D - P(t))]}{\partial t}.$$

Thus, following Ichimura and Newey (2022), if we find an $\alpha(\delta)$ such that:

$$\begin{aligned}
&\frac{\partial \mathbb{E}[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P(t))\beta_P(1, y) - DMTR_1)P(t)\mathbf{1}\{C > y\}]}{\partial t} \\
&= -\frac{\partial \mathbb{E}[\alpha(\delta)CZ(D - P(t))]}{\partial t},
\end{aligned} \tag{B.17}$$

then we will get (B.16).

Furthermore, from (B.9), we can see that

$$\frac{\partial \mathbb{E}[CZ(D - P(t))]}{\partial t} = \mathbb{E} \left[\frac{\partial CZ(D - P(t))}{\partial t} \right] = \mathbb{E} \left[a(\delta) \frac{\partial P(t)}{\partial t} \right] = \frac{\partial \mathbb{E}[a(\delta)P(t)]}{\partial t} \tag{B.18}$$

where $a(\delta) = CZ$.

From (B.15) and (B.18) combined with (B.17):

$$\frac{\partial \mathbb{E}[\tilde{b}(\delta)P(t)]}{\partial t} = -\frac{\partial \mathbb{E}[\alpha(\delta)a(\delta)P(t)]}{\partial t} \tag{B.19}$$

The equality in Equation (B.19) will be satisfied if it holds for every t . Since $P(t)$ is in the space of possible propensity scores, the condition will be satisfied if it holds for any P in the space of possible propensity scores. Thus:

$$\mathbb{E}[\tilde{b}(\delta)P] = -\mathbb{E}[\alpha(\delta)a(\delta)P] \tag{B.20}$$

or, equivalently,

$$0 = \mathbb{E}[\{\tilde{b}(\delta) + \alpha(\delta)a(\delta)\}P] = \mathbb{E} \left[(-a(\delta)) \left\{ \frac{-\tilde{b}(\delta)}{a(\delta)} - \alpha(\delta) \right\} P \right] \tag{B.21}$$

Thus, as in Ichimura and Newey (2022), the $\alpha(\delta)$ that minimizes $\mathbb{E} \left[(-a(\delta)) \left\{ \frac{-\tilde{b}(\delta)}{a(\delta)} - \alpha(\delta) \right\}^2 \right]$ satisfies

$$\phi(\delta, P, \alpha) = \alpha(\delta)CZ(D - P).$$

Combining this result with the previous display, the Influence Function of $DMTR_1$ is given by

$$\begin{aligned} IF_{DMTR_1} &= [-\mathbb{E}[P\mathbf{1}\{C > y\}]]^{-1} \left[(\Lambda'(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y) - DMTR_1)P\mathbf{1}\{C > y\} \right. \\ &\quad + \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y))P\mathbf{1}\{C > y\}]IF_{\beta_0(1, y)} \\ &\quad + \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y))P\mathbf{1}\{C > y\}]C]IF_{\beta_C(1, y)} \\ &\quad + \mathbb{E}[\Lambda''(\beta_0(1, y) + \beta_C(1, y)C + \beta_P(1, y)P)\beta_P(1, y))P\mathbf{1}\{C > y\}]P]IF_{\beta_P(1, y)} \\ &\quad \left. + \phi(\delta, P, \alpha) \right] \end{aligned} \quad (\text{B.22})$$

The influence function will depend on the influence functions of the estimated coefficients. Their influence function could be derived similarly to the nonparametric effect of P on $DMTR_1$, but using Equations (B.9)-(B.12). We could then use a similar logic as we did following Ichimura and Newey (2022) and a form of Riezs representation. That would fully complete the representation, implying that our previous claim holds.

Since $DMTE = DMTR_1 - DMTR_0$, we can then say that:

$$IF_{DMTE} = IF_{DMTR_1} - IF_{DMTR_0} \quad (\text{B.23})$$

where IF_{DMTR_0} can be derived analogously to IF_{DMTR_1} .

Then, we know that

$$\sqrt{N}(\widehat{DMTE} - DMTE) = \frac{1}{\sqrt{n}} \sum_i^n IF_{DMTE}(\delta_i) + o_p(1) \xrightarrow{d} N(0, Var(DMTE))$$

and

$$Var(DMTE) = \mathbb{E}[IF_{DMTE}^2],$$

where convergence is due to standard results on influence functions (van der Vaart and Wellner, 1996; van der Vaart, 1998).

Following Frandsen (2015) and our influence function calculations, we can recover the asymptotic distribution of the $QMT\mathbb{E}[\tau, p]$. The $QMT R_d$ as quantiles have the following standard and known influence functions:

$$IF_{QMT R_d(\tau, p)} = -\frac{1\{DMTR_d^{-1}(\tau, p) > y\} - \tau}{\frac{\partial DMTR_d(DMTR_d^{-1}(\tau, p), p)}{\partial y}},$$

implying that $IF_{QMTE(\tau,p)} = IF_{QMTR_1(\tau,p)} - IF_{QMTR_0(\tau,p)}$,

$$\sqrt{N}(\widehat{QMTE} - QMTE) = \frac{1}{\sqrt{n}} \sum_i^n IF_{QMTE}(\delta_i) + o_p(1) \xrightarrow{d} N(0, Var(QMTE))$$

and

$$Var(QMTE) = \mathbb{E}[IF_{QMTE}^2],$$

where convergence is due to standard results on influence functions (van der Vaart and Wellner, 1996; van der Vaart, 1998).

By recalling that $MTE(p) = \int_0^1 QMTE(\tau, p) d\tau$ and the fact that we just provided asymptotic normality for $QMTE(\tau, p)$ we can recover the distribution of the $MTE(p)$ following Masten, Poirier and Zhang (2020).

At this point, it is worth being specific about the definition of Hadamard differentiability and how it connects to the QTE and the MTE.

Definition 1. Let $\phi : D \rightarrow E$ where D, E are Banach spaces. Say ϕ is Hadamard differentiable at $\theta \in D$ if $\exists \phi'_\theta : D \rightarrow E$, $\forall h \in D$, if $t \rightarrow 0$, $\|h_t - h\| \rightarrow 0$, then:

$$\left\| \frac{\phi(\theta + th_t) - \phi(\theta)}{t} - \phi'_\theta(h) \right\|_E \rightarrow 0$$

In our context, we set $D = \mathcal{C}([0, 1], [0, 1])$ and $E = \mathcal{R}$, i.e., D is the space of continuous functions where the first component refers to τ and the second one to v . Then, we know that $\frac{QMTE(\theta_1 + th_{1t}, \theta_2 + th_{2t}) - QMTE(\theta_1, \theta_2)}{t} \xrightarrow{|||}_R QMTE'_{\theta_1, \theta_2}(h_1, h_2)$ and $\frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} \xrightarrow{|||}_R QMTE'_{\theta_2}(h_2)$, where $|||_R$ denotes the norm of convergence. Furthermore, we have that

$$\frac{MTE(\theta_2 + th_{2t}) - MTE(\theta_2)}{t} = \int_0^1 \frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} d\tau,$$

which, under the conditions for the dominated convergence theorem, implies that

$$\begin{aligned} \frac{MTE(\theta_2 + th_{2t}) - MTE(\theta_2)}{t} &= \int_0^1 \frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} d\tau \\ &\rightarrow \int_0^1 QMTE'_{\theta_2}(h_2) d\tau := MTE'_{\theta_2}(h_2). \end{aligned}$$

Consequently, the MTE is Hadamard differentiable, and we can apply the functional delta method again to get the asymptotic Gaussian distribution of the MTE.

Furthermore, by the chain rule of influence functions,

$$IF_{MTE(p)} = \int_0^1 IF_{QMTE(\tau,p)} d\tau$$

Although relevant to show the results for the MTE , in our text, we also focus on the $RMTE(v)$ to avoid additional support assumptions. Thus, we can similarly derive the

asymptotic results for the RMTE, since $RMTE(v) = -\int_0^{\gamma_C} DMTE(y, v) dy$. In the context of the RMTE, we set $D = \mathcal{C}([0, \gamma], [0, 1])$ and $E = \mathcal{R}$, i.e., D is the space of continuous functions where the first component refers to y , $\gamma = \min\{\gamma_1, \gamma_0\}$ and the second one to v . Then, we know that $\frac{DMTE(\theta_1 + th_{1t}, \theta_2 + th_{2t}) - DMTE(\theta_1, \theta_2)}{t} \rightarrow_{|||_R} DMTE'_{\theta_1, \theta_2}(h_1, h_2)$ and $\frac{DMTE(\tau, \theta_2 + th_{2t}) - DMTE(y, \theta_2)}{t} \rightarrow_{|||_R} DMTE'_{\theta_2}(h_2)$, where $|||_R$ denotes the norm of convergence. Furthermore, we have that

$$\frac{RMTE(\theta_2 + th_{2t}) - RMTE(\theta_2)}{t} = - \int_0^{\gamma_C} \frac{DMTE(y, \theta_2 + th_{2t}) - DMTE(y, \theta_2)}{t} dy,$$

which, under the conditions for the dominated convergence theorem, implies that

$$\begin{aligned} \frac{RMTE(\theta_2 + th_{2t}) - RMTE(\theta_2)}{t} &= - \int_0^{\gamma_C} \frac{DMTE(y, \theta_2 + th_{2t}) - DMTE(y, \theta_2)}{t} dy \\ &\rightarrow - \int_0^{\gamma_C} DMTE'_{\theta_2}(h_2) dy := RMTE'_{\theta_2}(h_2). \end{aligned}$$

Consequently, the RMTE is Hadamard differentiable, and we can apply the functional delta method to get the asymptotic Gaussian distribution.

Furthermore, by the chain rule of influence functions,

$$IF_{RMTE(p)} = - \int_0^{\gamma_C} IF_{DMTE(y, p)} dy.$$

B.4 Proof of Theorem 4.2: Validity of the Weighted Bootstrap

We generate $\{V_i, i = 1, \dots, n\}$ as a sequence of independent and identically distributed non-negative random variables with mean one, variance one, and finite third moment (e.g., $V_i \sim \text{Exp}(1)$).

Based on this we estimate the propensity score by minimizing the weighted version of the standard series minimization criteria,

$$\hat{\theta}^{fs,*} = \arg \min_{\theta^{fs} \in \Theta^{fs}} n^{-1} \sum_{i=1}^n V_i (D_i - \alpha_0 - X_i' \alpha_X - C_i \alpha_C - \psi^L(Z_i)' \alpha_Z)^2 \quad (\text{B.24})$$

where $\hat{\theta}^{fs,*} = (\hat{\alpha}_0^*, \hat{\alpha}_X^{*,*}, \hat{\alpha}_C^*, \hat{\alpha}_Z^*)'$ and thus \hat{P}_i^* is a function of $\hat{\theta}^{fs,*}$. Note then that by Corollary 3 in Ma and Kosorok (2005b) or Corollary 3.2.3 and Theorem 3.2.5 of Wellner et al. (2013), \hat{P}_i^* converges to P_i .

For the estimation of the β s, let:

$$Ln^*(\beta_{1,y}, \hat{P}) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i V_i W_{y,1,i} \log[\Lambda(\beta_{1,y} \hat{H}_i)] + V_i (1 - W_{y,1,i}) \log[1 - \Lambda(\beta_{1,y} \hat{H}_i)] \quad (\text{B.25})$$

And,

$$Ln^*(\beta_{1,y}, P) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i V_i W_{y,1,i} \log[\Lambda(\beta_{1,y} H_i)] + V_i (1 - W_{y,1,i}) \log[1 - \Lambda(\beta_{1,y} H_i)] \quad (\text{B.26})$$

By a similar display as in the unweighted case, we know that:

$$\sup_{\beta_{1,y}} |Ln^*(\beta_{1,y}, \hat{P}) - Ln^*(\beta_{1,y}, P)| = o_p(1)$$

Where $Ln^*(\beta_{1,y}, P)$ is a standard, but weighted parametric likelihood and thus by a second application of Corollary 3 in Ma and Kosorok (2005b) or Corollary 3.2.3 and Theorem 3.2.5 of Wellner et al. (2013), $\hat{\beta}_{1,y}^*$ converges to $\beta_{1,y}$.

It then remains to show asymptotic normality of $\hat{\beta}_{1,y}^*$ and then of $DMTR^*$. Since the V_i are independent of everything and $Var(V_i) = 1$, we have $\sqrt{N}(\hat{\beta}_{1,y}^* - \beta_{1,y})$ is asymptotically normal as long as a version of Assumption B.3 holds with \hat{P}_i^* .

Furthermore, since V_i is independent of everything, similar calculations of the influence functions for $DMTE$, $QMTE$, and $RMTE$ can be provided to obtain asymptotic normality of the bootstrapped versions of these. Alternatively, one can note that $DMTE$, $QMTE$, and $RMTE$ are all continuous and Hadamard differentiable functions of $\beta_{1,y}, P_i$, with \hat{P}^* slower than root- N via a bootstrap version of Assumption B.3 and thus functional delta methods and continuous mapping theorems hold.

C Constructing the Dataset

In this appendix, we summarize Appendix I.2 of Possebom (2022), which provides a detailed explanation of how the dataset used in our empirical application was constructed. We explain the specific crime types included in our sample, the classification algorithms used to define which defendants were punished, and the fuzzy matching algorithm used to define which defendants recidivate.

The final dataset was created from four initial datasets.

1. CPOPG (“Consulta de Processos de Primeiro Grau”): It contains information about all criminal cases in the Justice Court System in the State of São Paulo (TJ-SP) between 2010 and 2019.
2. CJPG (“Consulta de Julgados de Primeiro Grau”): It contains information about the last decision made by a trial judge in all criminal cases in TJ-SP between 2010 and 2019.
3. CPOSG (“Consulta de Processos de Segundo Grau”): It contains information about all appealing criminal cases in TJ-SP between 2010 and 2019.

Starting from the CPOPG dataset, we implement the following steps.

1. We only keep cases that are currently in the Appeals Court, closed, or whose status is empty. Those cases are already associated with a trial judge’s sentence.
2. We only keep cases whose crime types are associated with sentences that must be less than four years of incarceration.
3. We only keep cases that aim to analyze whether a defendant is guilty or not.
4. We only keep cases that were randomly assigned to trial judges.
5. We only keep cases whose starting date is after January 1st, 2010.

After these steps, our dataset contains 98,552 cases. We then merged it with the CJPG dataset using cases’ id codes. Since some cases do not have id codes, our dataset now contains 98,422 cases.

After this step, we randomly select 35 cases per year (2010-2019) for manual classification. We manually classify them into five categories: “defendant died during the trial”, “defendant is guilty”, “defendant accepted a non-prosecution agreement” (“transação penal” in Portuguese), “case was dismissed” (“processo suspenso” in Portuguese) and “defendant was acquitted”. Since some sentences are missing or incomplete, we are able to manually classify only 325 sentences.

Now, we use those 325 manually classified cases to train a classification algorithm. To do so, we divide them into a training sample (216 cases) and a validation sample (109 sentences).

First, we design an algorithm to identify which defendants died during the trial. To do so, we check whether the sentence contains any reference to the first paragraph of Article 107 from the Brazilian Criminal Code. This deterministic algorithm perfectly classifies cases into the category “defendant died during the trial”.

Second, we design an algorithm to identify which cases were dismissed. To do so, we check whether the sentence contains any reference to Article 89 in Law n. 9099/95. This deterministic algorithm correctly classifies 98% of the cases into the category “case was dismissed”.

Third, we design an algorithm to identify which defendants accepted a non-prosecution agreement. To do so, we check whether the sentence contains any expression connected to a non-prosecution agreement. This deterministic algorithm correctly classified almost all the cases into the category “defendant accepted a non-prosecution agreement”, making only three mistakes.

Finally, we design an algorithm to classify the remaining cases into two categories: “defendant is guilty” and “defendant was acquitted”. To do so, we define a bag of words that were found to be strong signals of acquittal and guilt when manually classifying the cases in our samples. we then count how many times each one of those expressions appears in each sentence, and we normalize those counts to be between 0 and 1.

Using the normalized counts, we train an L1-Regularized Logistic Regression using our training sample. We then validate this algorithm using our validation sample and find that it correctly classifies 98.8% of the cases. Given this high success rate, we use the L1-Regularized Logistic Regression algorithm to define the treatment variable in our full sample.

Having designed the above algorithm, we use it to define the trial judge’s treatment variable T in the full sample. First, we find which defendants died during their trials and drop them from my sample. We then use the second and third algorithms to define which cases were dismissed and which cases are associated with a non-prosecution agreement. Moreover, we use the trained L1-regularized Logistic Regression algorithm to classify the remaining cases into the categories “defendant is guilty” and “defendant was acquitted”. Finally, we combine the categories “defendant was acquitted” and “case was dismissed” into the untreated group (“not punished”, $T = 0$) and the categories “defendant accepted a non-prosecution agreement” and “defendant is guilty” into the treated group (“punished”, $T = 1$). At the end, our dataset contains 96,225 cases.

Now, we merge our current dataset with the CPOSG dataset using each case’s id code. When merging these datasets, we create an indicator variable that denotes which cases went to the Appeals Court, i.e., which cases were matched. We then randomly select 50 cases per year for manual classification (2010-2019) and divide them into three categories: “cases that went

to the Appeals Court, but were immediately returned due to bureaucratic errors”, “cases whose trial judge’s sentences were affirmed” and “cases whose trial judge’s sentences were reversed”.

Now, we use those 500 manually classified cases to train a classification algorithm. To do so, we divide them into a training sample (300 cases) and a validation sample (200 sentences).

First, we design an algorithm to identify which cases went to the Appeals Court but were immediately returned. To do so, we simply check whether the Appeals Court’s **decision** is empty.

Finally, we design an algorithm to classify the non-empty cases into two categories: “cases whose trial judge’s sentences were affirmed” and “cases whose trial judge’s sentences were reversed”. To do so, we define a bag of words that were found to be strong signals of sentence reversal when manually classifying the cases in our sample. We then count how many times each one of those expressions appears in each sentence, and we normalize those counts to be between 0 and 1.

Using the normalized counts, we train an L1-Regularized Logistic Regression using our training dataset. We then validate this algorithm using our validation sample and find that it correctly classifies 96.2% of the cases. Given this high success rate, we use the L1-Regularized Logistic Regression to define the treatment variable in our full sample.

Having designed the above algorithms, we use them to define the final treatment variable D in the full sample. First, we set $D = T$ if a case did not go to the Appeals Court or if a case went to the Appeals Court, but was immediately returned. Second, we use the trained L1-Regularized Logistic Regression algorithm to classify the remaining cases into the categories “reversed trial judge’s sentence” and “affirmed trial judge’s sentence”. We, then, set $D = T$ if the trial judge’s sentence was affirmed and $D = 1 - T$ if the trial judge’s sentence was reversed. Moreover, we also drop the cases whose dates (starting date, trial judge’s sentence date and Appeal Court’s decision date) are not appropriately ordered. At the end, our dataset contains 95,119 cases.

Now, our goal is to find the defendants’ names in each case. To do so, we use the variable **parties** from the CPOPG dataset and search for names listed as defendants. Finally, we delete names that are not a person’s name — such as district attorney offices, public defender offices and “unknown author”. Our sample now contains 103,423 case-defendant pairs.

Furthermore, we repeat the steps in the last paragraph to find defendants’ names in a dataset that contains all cases from the CPOPG dataset, including cases that are still open and cases with severe crimes. This dataset contains 1,027,120 case-defendants pairs.

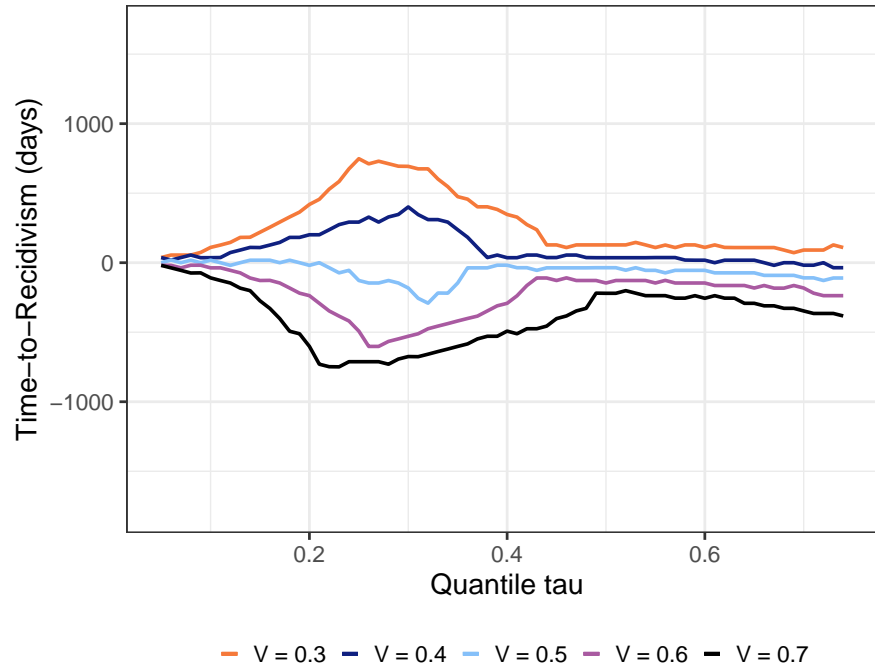
Now, we use these two datasets to define our outcome variable (Y = “time to recidivism”). A defendant i in a case j in the smaller dataset recidivates if and only if defendant i ’s full name appears in a case \bar{j} in the larger dataset. To match defendants’ names across cases, we use the Jaro–Winkler similarity metric and we define a match if the similarity between full names in

two different cases is greater than or equal to 0.95. If we find a match, we define the outcome variable as the time difference between the second case's start date and the first case's final date.

Finally, we delete the case-defendant pairs whose cases started in 2018 and 2019. Consequently, our dataset contains 51,731 case-defendants pairs.

D Additional Empirical Results

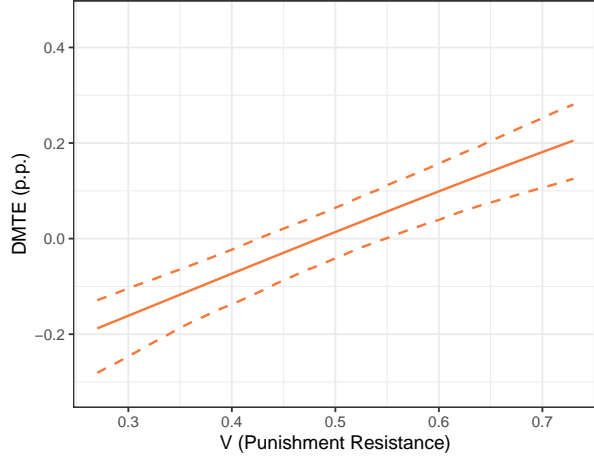
Figure D.1: $QMTE(\cdot, v)$ for $v \in \{.3, .4, \dots, .7\}$



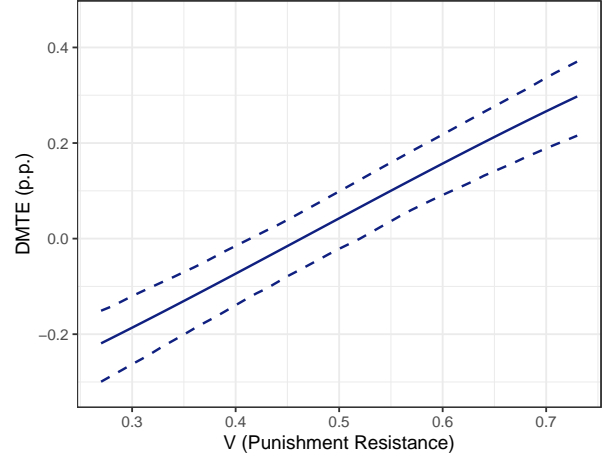
Notes: Solid lines are the point estimates for the average $QMTE(\cdot, v)$ functions indicated in the legend. These results are based on Corollary 4.1.

D.1 Confidence Intervals for DMTE, QMTE and RMTE functions

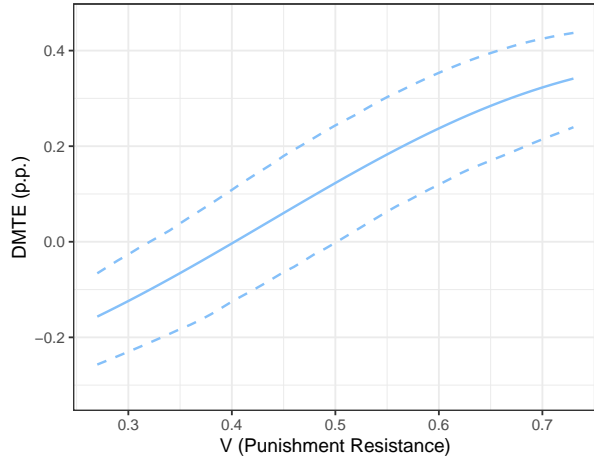
Figure D.2: 90%-Confidence Intervals for $DMTE(y, \cdot)$ for $y \in \{1, 2, 3, 4\}$



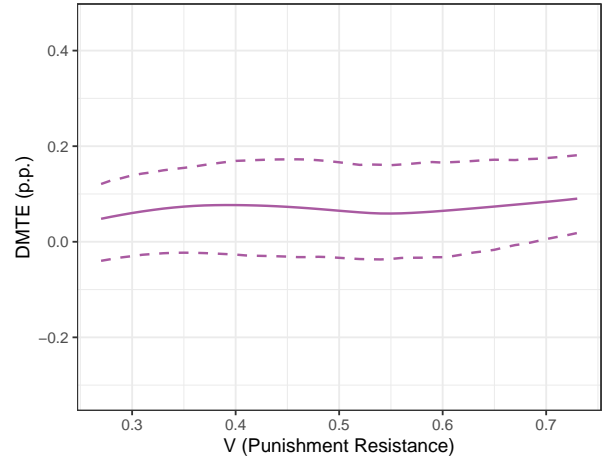
(a) $DMTE(1, \cdot)$



(b) $DMTE(2, \cdot)$



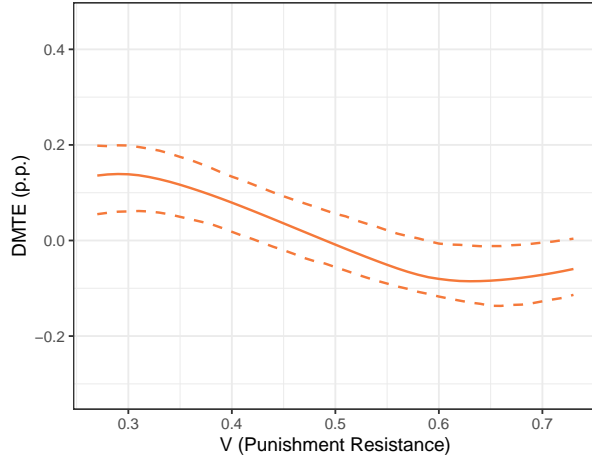
(c) $DMTE(3, \cdot)$



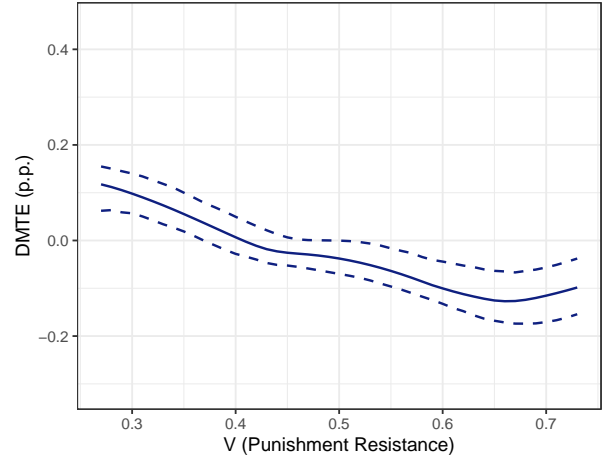
(d) $DMTE(4, \cdot)$

Notes: Solid lines are the point estimates for the average $DMTE(y, \cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clustered at the court district level (Subsection 4.2).

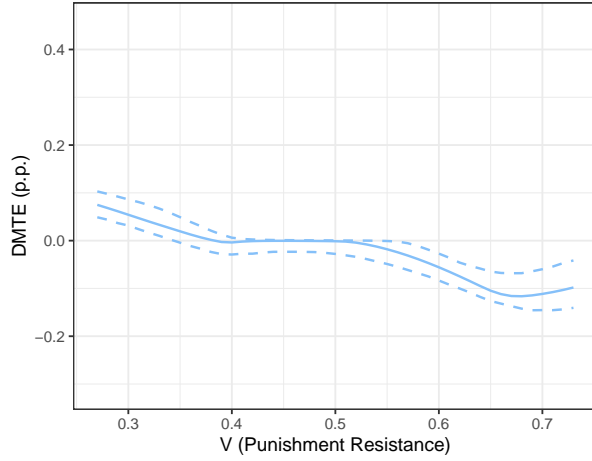
Figure D.3: 90%-Confidence Intervals for $DMTE(y, \cdot)$ for $y \in \{5, 6, 7, 8\}$



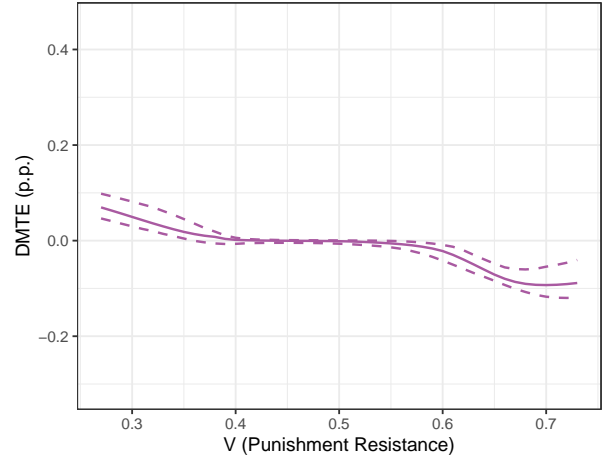
(a) $DMTE(5, \cdot)$



(b) $DMTE(6, \cdot)$



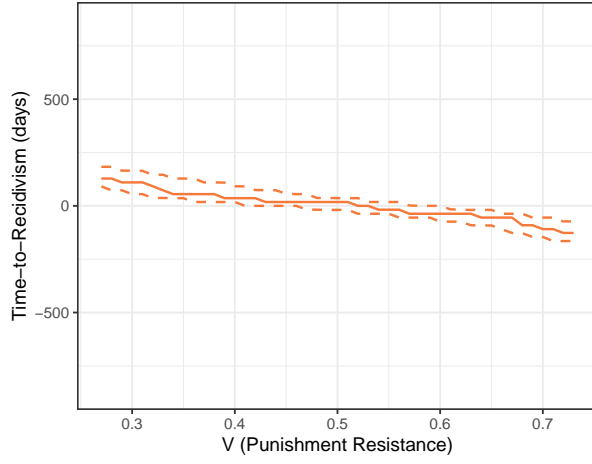
(c) $DMTE(7, \cdot)$



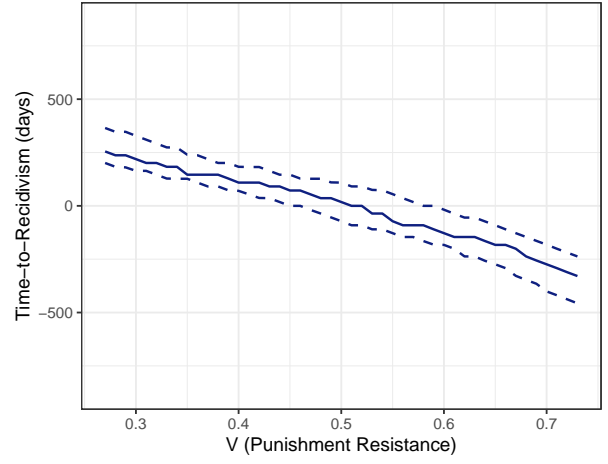
(d) $DMTE(8, \cdot)$

Notes: Solid lines are the point estimates for the average $DMTE(y, \cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clustered at the court district level (Subsection 4.2).

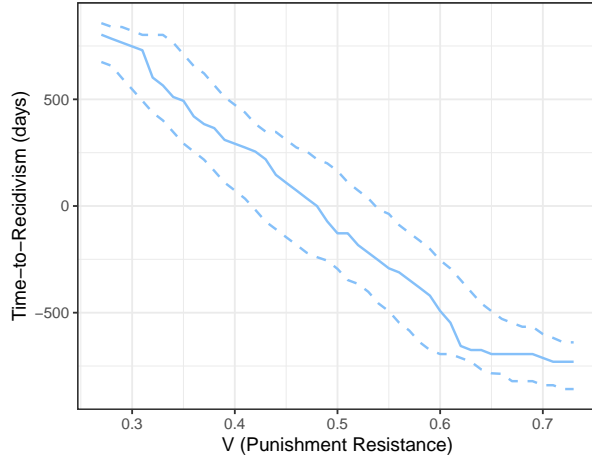
Figure D.4: 90%-Confidence Intervals for $QMTE(\tau, \cdot)$ for $\tau \in \{.10, .15, .25, .30\}$



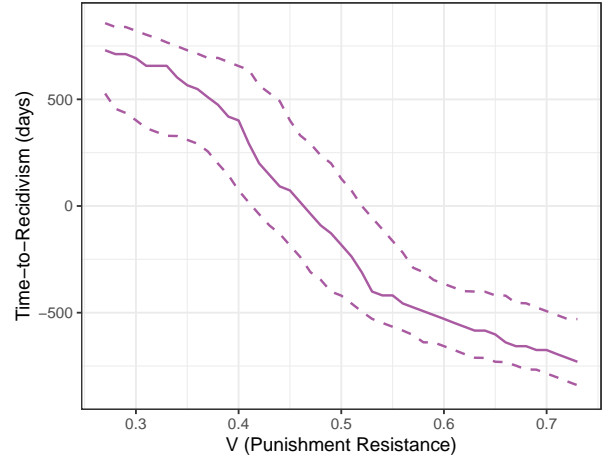
(a) $QMTE(.10, \cdot)$



(b) $QMTE(.15, \cdot)$



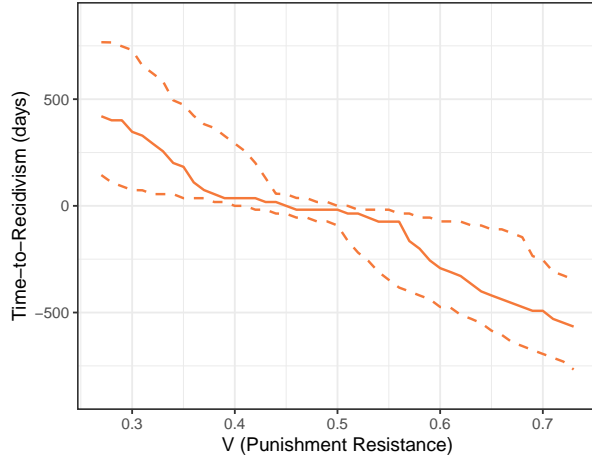
(c) $QMTE(.25, \cdot)$



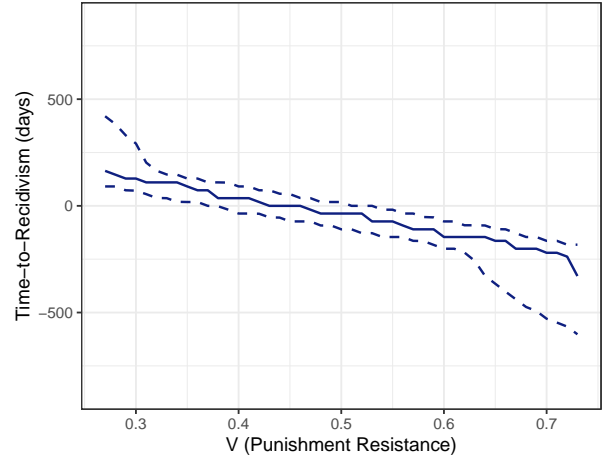
(d) $QMTE(.30, \cdot)$

Notes: Solid lines are the point estimates for the average $QMTE(\tau, \cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clustered at the court district level (Subsection 4.2).

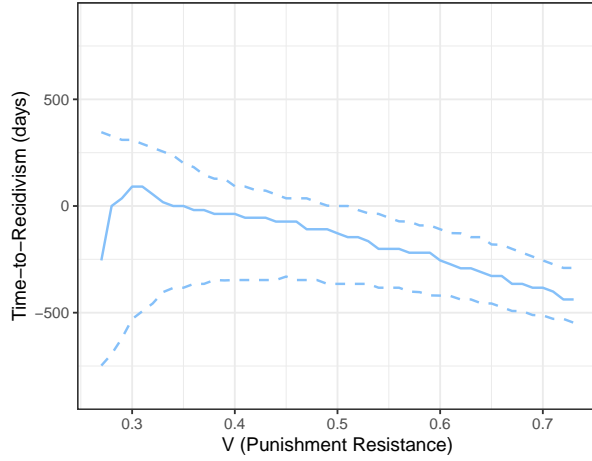
Figure D.5: 90%-Confidence Intervals for $QMTE(\tau, \cdot)$ for $\tau \in \{.40, .50, .75\}$ and $RMTE(\cdot)$



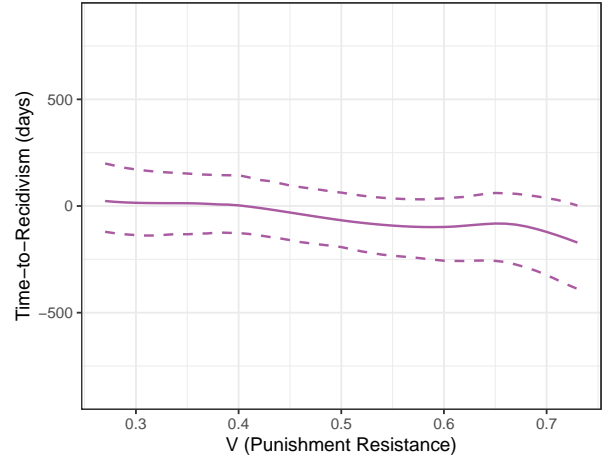
(a) $QMTE(.40, \cdot)$



(b) $QMTE(.50, \cdot)$



(c) $QMTE(.75, \cdot)$



(d) $RMTE(\cdot)$

Notes: Solid lines are the point estimates for the average $QMTE(\tau, \cdot)$ and $RMTE(\cdot)$ functions indicated in the caption of each subfigure. These results are based on Corollary 4.1. Moreover, point-wise 90%-confidence intervals are reported using dashed lines. These confidence intervals were computed using the weighted bootstrap clustered at the court district level (Subsection 4.2).

E Relevance of MTE for Duration Outcomes

In this appendix, we justify focusing on the marginal treatment effect (MTE) for duration outcomes using two arguments.

In Appendix E.1, we develop a theoretical model with a policymaker who selects a treatment assignment rule that minimizes the cost of recidivism for the target population of defendants.

In Appendix E.2, we provide a simple example where the treatment benefits most agents in our population. In this example, our proposed focus on quantile treatment effects for duration outcomes correctly highlights that this treatment benefits society. However, focusing on short-time horizons as usually done in the crime economics literature leads to the opposite conclusion.

E.1 Theoretical Justification of Relevance of MTE for Duration Outcomes

Following Kitagawa and Tetenov (2018), the policymaker has to choose a treatment rule that determines whether individuals with variables $W = \{Z, V, C\}$ in our target population will be assigned to the treatment group or the control group. The policymaker chooses non-randomized treatment rules that are described by decision sets $G \subset \mathcal{W}$, where \mathcal{W} is the support of W . These decision sets determine the group of individuals $\{W \in G\}$ to whom treatment is assigned. We denote the collection of candidate treatment rules by $\mathcal{G} = \{G \subset \mathcal{W}\}$.

The goal of the policymaker in our context is to select a treatment assignment rule that minimizes the cost of recidivism for the target population of defendants. Assuming that the policymaker discounts cost inter-temporally, she chooses the treatment rule that maximizes Y^* for each individual in the target population.

Specifically, we impose that the policymaker chooses the decision set $G \in \mathcal{G}$ that minimizes

$$K(G) := \mathbb{E} \left[\ln \left\{ b^{[Y^*(1) \cdot \mathbf{1}\{W \in G\} + Y^*(0) \cdot \mathbf{1}\{W \notin G\}]} \cdot k \right\} \right]$$

where $k \in \mathbb{R}_+$ is the fixed cost of recidivism and $b \in (0, 1)$ is the policymaker's discount rate. Rearranging the last equation, we find that

$$\begin{aligned} K(G) &= \ln \{b\} \cdot \mathbb{E} [Y^*(1) \cdot \mathbf{1}\{W \in G\} + Y^*(0) \cdot \mathbf{1}\{W \notin G\}] + \ln \{k\} \\ &= \ln \{b\} \cdot \mathbb{E} [(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}] + \ln \{b\} \cdot \mathbb{E} [Y^*(0)] + \ln \{k\} \end{aligned}$$

Consequently, the policymaker's problem is equivalent to

$$\max_{G \in \mathcal{G}} \mathbb{E} [(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}].$$

Moreover, note that

$$\begin{aligned} &\mathbb{E} [(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}] \\ &= \mathbb{E} [\mathbb{E} [(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\} | V, Z, C]] \end{aligned}$$

$$\begin{aligned}
& \text{by the Law of Iterated Expectations} \\
&= \mathbb{E} [\mathbb{E} [(Y^*(1) - Y^*(0)) | V, Z, C] \cdot \mathbf{1}\{W \in G\}] \\
&= \mathbb{E} [\mathbb{E} [(Y^*(1) - Y^*(0)) | V, C] \cdot \mathbf{1}\{W \in G\}] \\
& \quad \text{by Assumption 1} \\
&= \mathbb{E} [\mathbb{E} [(Y^*(1) - Y^*(0)) | V] \cdot \mathbf{1}\{W \in G\}] \\
& \quad \text{by Assumption 5} \\
&= \mathbb{E} [MTE(V) \cdot \mathbf{1}\{W \in G\}].
\end{aligned}$$

Therefore, the policymaker's problem is equivalent to

$$\max_{G \in \mathcal{G}} \mathbb{E} [MTE(V) \cdot \mathbf{1}\{W \in G\}],$$

implying that focusing on the MTE of duration outcomes is relevant when the policymaker wishes to minimize the cost of recidivism over time.

E.2 Illustrating the Relevance of Duration Outcomes

When analyzing the impact of judicial decisions on recidivism, many authors (Agan et al., 2023; Bhuller et al., 2019; Giles, 2021; Huttunen et al., 2020; Klaassen, 2021; Possebom, 2022) focus on a short time horizon, using a small set of outcome variables that indicate whether the defendant recidivated within a pre-specified number of years. In this paper, we advocate for moving beyond this short time horizon and focusing on quantile or average treatment effects of duration outcomes.

In this appendix, we illustrate why focusing on duration outcomes may provide more information than the standard approach in the empirical literature in crime economics. To do so, we abstract from the MTE heterogeneity (variable V) and focus exclusively on the heterogeneity arising from the distribution of the potential outcomes $(Y^*(0), Y^*(1))$.

We illustrate the relevance of quantile and average treatment effects of duration outcomes by analyzing a simple example with discrete random variables. In this example, focusing on short-term outcomes or long-term quantile treatment effects lead to different conclusions about our policy of interest.

We denote potential time-to-recidivism by $Y^*(0)$ and $Y^*(1)$ and measure it in years. Table E.1 shows the joint probability mass function of $(Y^*(0), Y^*(1))$ and their marginal distributions.

Note that, in this example, our judicial decision benefits most defendants. For instance, this treatment strictly increases time-to-recidivism for 50% of the defendants ($Y^*(0) < Y^*(1)$). Moreover, only 20% of the defendants are harmed by this treatment ($Y^*(0) > Y^*(1)$).

However, a short-time horizon analysis would conclude that this treatment is harmful. For example, this treatment increases the probability of recidivism within one year by 5 p.p. and the

Table E.1: Joint Probability of $(Y^*(0), Y^*(1))$ and their Marginal Distributions

$\mathbb{P}[Y^*(0) = \cdot, Y^*(1) = \cdot]$		$Y^*(0) =$					$\mathbb{P}[Y^*(1) = \cdot]$
		1	2	3	10	20	
$Y^*(1) =$	1	.10	0	.10	0	0	.20
	2	0	.10	.10	0	0	.20
	3	0	0	0	0	0	0
	10	0	0	0	.10	0	.10
	20	.05	.05	0	.40	0	.50
$\mathbb{P}[Y^*(0) = \cdot]$.15	.15	.20	.50	0	1

Note: The last column reports the marginal distribution of $Y^*(1)$. The last row reports the marginal distribution of $Y^*(0)$. The cells in the center of the table report the joint distribution of $(Y^*(0), Y^*(1))$.

probability of recidivism within two years by 10 p.p, i.e., $\mathbb{P}[Y^*(1) \leq 1] - \mathbb{P}[Y^*(0) \leq 1] = 0.05$ and $\mathbb{P}[Y^*(1) \leq 2] - \mathbb{P}[Y^*(0) \leq 2] = 0.1$.

Differently from the standard empirical analysis, we advocate for focusing on quantile and average treatment effects of duration outcomes. For example, the Quantile Treatment Effect on the Median is equal to seven years because the median of $Y^*(1)$ equal ten years and the median of $Y^*(0)$ equals three years. Moreover, the average treatment effect equals 5.55 years in this example.

Therefore, our proposed analysis would correctly highlight that this treatment benefits at least some agents in our society.

F Identification without Restrictions on Censoring

In this appendix, we focus on which parameters can be point-identified when we do not impose any restriction on the relationship between the censoring variable and the potential outcomes. To compensate for not imposing Assumption 5 nor Assumption G.1, we need to allow the *DMTR* function to depend on the censoring variable.

Specifically, our target parameter is given by:

$$DMTR_d(y, v, c) := \mathbb{P}[Y^*(d) \leq y | V = v, C = c]$$

for any $d \in \{0, 1\}$, $y < \gamma_C$, $v \in [0, 1]$ and $c \in \mathcal{C}$. Note that our target parameter is interpretable as a conditional distributional marginal treatment response. In particular, the censoring variable C acts similarly to a covariate in the standard MTE analysis (Carneiro et al., 2011).

In our empirical application, conditioning on the censoring variable is equivalent to conditioning on the defendant cohort or time fixed effects. Considering that most studies about judicial decisions (Agan et al., 2023; Bhuller et al., 2019; Huttunen et al., 2020; Klaassen, 2021) condition on district-by-time fixed effects, they identify the conditional *DMTR* function for a pre-specified value of y . In this appendix, we discuss how to extend their analysis to consider conditional quantile marginal treatment effects and marginal treatment effects (Remark 4).

To point-identify the conditional *DMTR* function, we eliminate Assumptions 5 and G.1 and impose Assumptions 1-4 only.

Proposition F.1. *If Assumptions 1-4 hold, then*

$$DMTR_d(y, p, y + \delta) = (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = p, C = y + \delta]}{\partial v}$$

for any $d \in \{0, 1\}$, $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

Remark 4. A direct consequence of Proposition F.1 is the identification of the quantile marginal treatment response function $QMTR_d(\tau, p, y + \delta)$ conditional on the censoring variable for any $\tau \in (0, \bar{\tau}_d(p, y + \delta))$, where $\bar{\tau}_d(p, y + \delta) := DMTR_d(\gamma_C, p, y + \delta)$. Additionally, if we impose Assumptions 6 and 7, then we straight-forwardly identify the MTE function conditional on the censoring variable.

Remark 5. The comparison between Propositions 3.1 and F.1 illustrate the identifying power of Assumption 5. It allows us to combine multiple values of the censoring variable to identify a single point of the *DMTR* function through the integral of $\frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = p, C = y + \delta]}{\partial v}$ over different values of δ . In our empirical application, it means that we can combine multiple defendant cohorts to identify a single evaluation point in the *DMTR* function.

Proof. For brevity, we show the proof of Proposition F.1 when $d = 1$.

Fix $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$. Note that

$$\begin{aligned}
& \mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta] \\
&= \mathbb{E}[\mathbf{1}\{Y \leq y\} \mathbf{1}\{P(Z, C) \geq V\} | P(Z, C) = v, C = y + \delta] \\
&\quad \text{by (3.1)} \\
&= \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} \mathbf{1}\{p \geq V\} | P(Z, y + \delta) = p, C = y + \delta] \\
&\quad \text{because } Y_1^* \text{ is not censored when } C > y \\
&= \int_0^1 \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} \mathbf{1}\{p \geq v\} | P(Z, y + \delta) = p, C = y + \delta, V = v] dv \\
&\quad \text{by the Law of Iterated Expectations and Assumption 3} \\
&= \int_0^1 \mathbf{1}\{p \geq v\} \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} | P(Z, y + \delta) = p, C = y + \delta, V = v] dv \\
&= \int_0^p \mathbb{E}[\mathbf{1}\{Y^*(1) \leq y\} | P(Z, y + \delta) = p, C = y + \delta, V = v] dv \\
&= \int_0^p \mathbb{P}[Y^*(1) \leq y | C = y + \delta, V = v] dv \\
&\quad \text{by Assumption 1.}
\end{aligned}$$

Consequently, the Leibniz Integral Rule implies that

$$\begin{aligned}
\frac{\partial \mathbb{P}[Y \leq y, D = 1 | P(Z, C) = p, C = y + \delta]}{\partial v} &= \mathbb{P}[Y^*(1) \leq y | C = y + \delta, V = p] \\
&= DMTR_1(y, p, y + \delta).
\end{aligned}$$

We can prove the same result for $d = 0$ analogously. ■

G Partial Identification Strategies

In some empirical applications, Assumption 5 may be too strong, while in others, it may be plausible. Since this is very context-specific, it is worth coming up with alternative identification strategies that accommodate dependent censoring mechanisms. In this appendix, we discuss two alternative assumptions that restrict the dependence between the censoring variable and the latent heterogeneity, but not to the point of imposing censoring independence. Section G.1 imposes that potential outcomes are negatively regression-dependent on the censoring variable, while Section G.2 intuitively imposes that the censoring problem is not severe.

G.1 Partial Identification under Regression Dependence

In this subsection, we impose that potential outcomes are negatively regression-dependent on the censoring variable. This alternative assumption restricts the relationship between the latent heterogeneity, the censoring variable, and the potential outcomes.²⁵

Assumption G.1 (Regression Dependence). *Conditional on V , the potential outcomes are negatively regression dependent on the censoring variable, i.e., $\mathbb{P}[Y^*(d) \leq y | C = \tilde{c}, V = v] \geq \mathbb{P}[Y^*(d) \leq y | C = c, V = v]$ for any $d \in \{0, 1\}$, any $v \in (0, 1)$ and any $(c, \tilde{c}) \in \mathcal{C}^2$ such that $c \leq \tilde{c}$.*

In our empirical application, Assumption G.1 imposes that the potential outcomes of more recent cases first-order stochastically dominate the potential outcomes of older cases. Intuitively, this restriction imposes that defendants are committing fewer crimes over time and is plausible given that the state of São Paulo became safer during our sampling period.²⁶

To derive bounds around the *DMTR* functions, define the following auxiliary quantities:

$$\begin{aligned} LB_d(y, v, \delta) &:= \mathbb{P}(y + \delta \leq C) \cdot (2d - 1) \cdot \gamma_d(y, v, y + \delta) \\ UB_d(y, v, \delta) &:= \mathbb{P}(C \leq y) + \mathbb{P}(y + \delta \leq C) \\ &\quad + \mathbb{P}(y \leq C \leq y + \delta) \cdot (2d - 1) \cdot \gamma_d(y, v, y + \delta), \end{aligned}$$

where $\delta \in \mathbb{R}_{++}$. The next proposition describes the bounds around the *DMTR* functions when potential outcomes are negatively regression-dependent on the censoring variable.

Proposition G.1. *Suppose that Assumptions 1-4 and G.1 hold. Then,*

$$DMTR_d(y, v) \in \left[\max_{\delta \in \mathcal{D}} LB_d(y, v, \delta), \min_{\delta \in \mathcal{D}} UB_d(y, v, \delta) \right]$$

²⁵Related assumptions have been used by Chesher (2005), Jun, Pinkse and Xu (2011), and Kedagni and Mourifie (2014) in different contexts. For more information on the definition of regression dependence and other concepts of statistical dependence, see Lehmann (1966).

²⁶In a different empirical context, positive regression dependence may be more plausible than negative regression dependence. Similar bounds can be derived based on this alternative assumption.

for any $d \in \{0, 1\}$, $y < \gamma_C$ and $v \in \mathcal{P}$, where $\mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$.

Proof. See Appendix G.1.1. ■

First, one can see that the bounds in Proposition G.1 do not “collapse” to point identification when Assumption 5 holds. That is because the regression-dependence in Assumption G.1 is compatible with dependent censoring but does not directly restrict the amount of dependence between the potential outcomes and the censoring random variable. In other words, the nature of Assumption G.1 is different from Assumption 5 and does not constitute “continuous relaxations” of the independence assumption. On the other hand, Assumption G.1 allows us to exploit the information in $\gamma_d(y, v, y + \delta)$ even under dependent censoring because of the (stochastic) monotonicity in C . Since this monotonicity property holds for different values of C less than the \tilde{c} , we take the supremum and infimum over δ , so the bounds are tighter. The identification region will functionally depend on the propensity score (and thus the instrument), as reflected in the presence of $\gamma_d(y, v, y + \delta)$ in the bounds. This will determine the shape of it. Furthermore, the bounds’ length depends on the proportion of censored observations close to the value of the particular y , reflecting that regions with heavier censoring data tend to have wider bounds.

From the partial identification of the $DMTR_d(y, v)$ functions, it also follows the partial identification of a range of $QMTE(\tau, v)$ across τ and the $RMTE(v)$, just like before. If one further imposes Assumptions 6 and 7, partial identification results for the MTE function will also follow. For these functions, though, it is important to ensure that the lower and upper bounds in Proposition G.1 are monotone in y , which can be enforced using a similar approach as in Manski and Molinari (2021). More specifically, for a grid of weakly increasing y ’s, if $\max_{\delta \in \mathcal{D}} LB_d(y_{k+1}, c, \delta) < \max_{\delta \in \mathcal{D}} LB_d(y_k, c, \delta)$, we can simply redefine $\max_{\delta \in \mathcal{D}} LB_d(y_{k+1}, c, \delta) = \max_{\delta \in \mathcal{D}} LB_d(y_k, c, \delta)$; the analogous is true for the upper bound. Alternatively, one can use the rearrangement procedure as in Chernozhukov et al. (2009). We state these results as corollaries for convenience.

Corollary G.1. *Suppose that Assumptions 1-4 and G.1 hold. Then,*

- (a) *$QMTE(\tau, v)$ is partially identified for any $v \in \mathcal{P}$ and $\tau \in (0, \bar{\tau}(v))$.*
- (b) *the $RMTE(v)$ function is partially-identified for any $v \in \mathcal{P}$.*

Corollary G.2. *If Assumptions 1-4, G.1, 6 and 7 hold, then $MTE(v)$ is partially identified for any $v \in \mathcal{P}$.*

Finally, the bounds in Proposition G.1 can be estimated using methods similar to the methods described in Subsection 4.2. The main difference between the estimators of the bounds and the point-estimators (Subsection 4.2) is that, when estimating the bounds, we take either the

maximum or the minimum over values of c in Step 5 instead of taking the mean. Consequently, these estimators will converge in probability to the bounds in Proposition G.1.

G.1.1 Proof of Proposition G.1

Fix $d \in \{0, 1\}$, $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

Note that Equations (A.1) and (A.2) imply that

$$\frac{\partial \mathbb{P}[Y \leq y, D = 1 | P(Z, C) = v, C = y + \delta]}{\partial v} = \mathbb{P}[Y^*(1) \leq y | C = y + \delta, V = v] \quad (\text{G.1})$$

and

$$\frac{\partial \mathbb{P}[Y \leq y, D = 0 | P(Z, C) = v, C = y + \delta]}{\partial v} = -\mathbb{P}[Y^*(0) \leq y | C = y + \delta, V = v] \quad (\text{G.2})$$

according to the Leibniz Integral Rule.

Combining the last two equations, we have that

$$\mathbb{P}[Y^*(d) \leq y | C = y + \delta, V = v] = (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v}. \quad (\text{G.3})$$

Moreover, observe that:

$$\begin{aligned} & \mathbb{P}[Y^*(d) \leq y | V = v] \\ &= \int \mathbb{P}[Y^*(d) \leq y | C = \tilde{c}, V = v] f_{C|V}(\tilde{c}|v) d\tilde{c} \\ & \quad \text{by the Law of Iterated Expectations} \\ &= \int \mathbb{P}[Y^*(d) \leq y | C = \tilde{c}, V = p] f_C(\tilde{c}) d\tilde{c} \\ & \quad \text{because } V \perp\!\!\!\perp C \text{ by Assumption 3} \\ &= \int_0^y \mathbb{P}[Y^*(d) \leq y | C = \tilde{c}, V = p] f_C(\tilde{c}) d\tilde{c} \\ & \quad + \int_y^{y+\delta} \mathbb{P}[Y^*(d) \leq y | C = \tilde{c}, V = p] f_C(\tilde{c}) d\tilde{c} \\ & \quad + \int_{y+\delta}^{+\infty} \mathbb{P}[Y^*(d) \leq y | C = \tilde{c}, V = p] f_C(\tilde{c}) d\tilde{c}, \end{aligned}$$

implying, by Assumption G.1, that

$$\begin{aligned} \mathbb{P}[Y^*(d) \leq y | V = p] &\leq \mathbb{P}(C \leq y) + \mathbb{P}(y + \delta \leq C) \\ &\quad + \mathbb{P}(y \leq C \leq y + \delta) \mathbb{P}[Y^*(d) \leq y | C = y + \delta, V = p] \end{aligned} \quad (\text{G.4})$$

and

$$\mathbb{P}[Y^*(d) \leq y | V = p] \geq \mathbb{P}(y + \delta \leq C) \mathbb{P}[Y^*(d) \leq y | C = y + \delta, V = p] \quad (\text{G.5})$$

Thus, combining Equations (G.4) and (G.5) with (G.3), we have that

$$DMTR_d(y, v)$$

$$\in \left[\begin{aligned} & \mathbb{P}(y + \delta \leq C) \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v}, \\ & \mathbb{P}(C \leq y) + \mathbb{P}(y + \delta \leq C) \\ & + \mathbb{P}(y \leq C \leq y + \delta) \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v} \end{aligned} \right].$$

Since the bounds above hold for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$\begin{aligned} & DMTR_d(y, v) \\ & \in \left[\max_{\delta \in \mathcal{D}} \left\{ \mathbb{P}(y + \delta \leq C) \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v} \right\}, \right. \\ & \quad \left. \min_{\delta \in \mathcal{D}} \left\{ \begin{aligned} & \mathbb{P}(C \leq y) + \mathbb{P}(y + \delta \leq C) + \mathbb{P}(y \leq C \leq y + \delta) \\ & \cdot (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v} \end{aligned} \right\} \right], \end{aligned}$$

where $\mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$.

G.2 Partial Identification under a Continuous Violation of Random Censoring

In this subsection, we impose that the conditional distribution of the potential outcomes given the censoring variable and the latent heterogeneity variable is close to the conditional distribution of the potential outcomes given only the latent heterogeneity variable.²⁷ Differently from Assumption G.1, the following assumption constitutes a “continuous relaxation” of censoring independence (Assumption 5).

Assumption G.2 (“Continuous Relaxation”). *The conditional distribution of the potential outcomes given the censoring variable and the latent heterogeneity variable is similar to the conditional distribution of the potential outcomes given only the latent heterogeneity variable i.e., there exists $\bar{B} \in \mathbb{R}_{++}$ such that*

$$|\mathbb{P}[Y^*(d) \leq y | C = y + \delta, V = v] - \mathbb{P}[Y^*(d) \leq y | V = v]| \leq \bar{B}$$

for any $y \in \mathcal{Y}$, $v \in [0, 1]$, $\delta \in \mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$ and $d \in \{0, 1\}$.

Using this assumption, we can derive bounds around the $DMTR$ functions.

Proposition G.2. *Suppose that Assumptions 1-4 and G.2 hold. Then,*

$$DMTR_d(y, v) \in \left[-\bar{B} + \left(2d - 1\right) \cdot \max_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta), \bar{B} + \left(2d - 1\right) \cdot \min_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta) \right]$$

for any $d \in \{0, 1\}$, $y < \gamma_C$ and $v \in \mathcal{P}$.

Proof. See Appendix G.2.1. ■

²⁷A similar assumption is used by Kline and Santos (2013) in a sample selection context.

First, differently from Proposition G.1, the bounds in Proposition G.2 “collapse” to point identification when Assumption 5 holds. Since our continuous relaxation of censoring independence holds for every value of C greater than y , we take the supremum and infimum over δ to tighten the bounds. The identification region will functionally depend on the propensity score (and thus the instrument), as reflected in the presence of $\gamma_d(y, v, y + \delta)$ in the bounds. This will determine the shape of it.

Furthermore, the bounds’ length depends on the choice of \bar{B} . We recommend choosing \bar{B} according to a breakdown analysis (Kline and Santos, 2013; Masten and Poirier, 2018). For example, the researcher may be particularly interested in $DMTE(\bar{y}, \cdot)$ for some value of $\bar{y} \in \mathcal{Y}$. If, based on Proposition 3.1, $DMTE(\bar{y}, v) \neq 0$ for some $v \in \mathcal{P}$, then the researcher can choose the smallest value of \bar{B} such that the bounds in Proposition G.2 contain the zero function. This value of \bar{B} is known as the breakdown point.²⁸

From the partial identification of the $DMTR_d(y, v)$ functions, it also follows the partial identification of a range of $QMTE(\tau, v)$ across τ and the $RMTE(v)$, just like before. If one further imposes Assumptions 6 and 7, partial identification results for the MTE function will also follow. For these functions, though, it is important to ensure that the lower and upper bounds in Proposition G.2 are monotone in y , which can be enforced using a similar approach as in Manski and Molinari (2021). Alternatively, one can use the rearrangement procedure as in Chernozhukov et al. (2009). We state these results as corollaries for convenience.

Corollary G.3. *Suppose that Assumptions 1-4 and G.2 hold. Then,*

- (a) *$QMTE(\tau, v)$ is partially identified for any $v \in \mathcal{P}$ and $\tau \in (0, \bar{\tau}(v))$.*
- (b) *the $RMTE(v)$ function is partially-identified for any $v \in \mathcal{P}$.*

Corollary G.4. *If Assumptions Assumptions 1-4, G.2, 6 and 7 hold, then $MTE(v)$ is partially identified for any $v \in \mathcal{P}$.*

Finally, the bounds in Proposition G.2 can be estimated using methods similar to the methods described in Subsection 4.2. The main difference between the estimators of the bounds and the point-estimators (Subsection 4.2) is that, when estimating the bounds, we take either the maximum or the minimum over values of c in Step 5 instead of taking the mean. Consequently, these estimators will converge in probability to the bounds in Proposition G.2.

G.2.1 Proof of Proposition G.2

Fix $d \in \{0, 1\}$, $y < \gamma_C$, $v \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

²⁸Conducting inference about the breakdown point is beyond the scope of this paper.

To derive the upper bound, observe that

$$\begin{aligned}
DMTR_d(y, v) &= \mathbb{P}[Y^*(d) \leq y | V = v] \\
&\quad \text{by definition} \\
&\leq \overline{B} + \mathbb{P}[Y^*(d) \leq y | C = y + \delta, V = v] \\
&\quad \text{according to Assumption \textcolor{red}{G.2}} \\
&= \overline{B} + (2d - 1) \cdot \frac{\partial \mathbb{P}[Y \leq y, D = d | P(Z, C) = v, C = y + \delta]}{\partial v} \\
&\quad \text{according to (\textcolor{red}{G.3})} \\
&= \overline{B} + (2d - 1) \cdot \gamma_d(y, v, y + \delta) \\
&\quad \text{by definition.}
\end{aligned}$$

Since the bounds above hold for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$DMTR_d(y, v) \leq \overline{B} + (2d - 1) \cdot \min_{\delta \in \mathcal{D}} \gamma_d(y, v, y + \delta).$$

We can derive the lower bound analogously.